Department of Computer Science **University of San Francisco**

Computer Science 673
Midterm 2 Review
Fall 2016

1. There is a river that you want to travel down by canoe. There are n trading posts along the river, you can rent a canoe at any trading post, paddle downstream, and return it at any trading post downstream. You want to find the cheapest way to get from the start to the end of the river, where:
   - For each pair of trading posts i, j, there is a cost function $c(i.j)$ to rent a canoe at post i and take it to post j
   - $c(i, j)$ may be $> c(i, k) + c(k, j)$
   - Can't go upstream: $c(i.j) = \infty$ when $i > j$
2. You have implemented disjoint sets using both path compression and union-by-rank. What is the largest possible tree depth you can get for a set of 16 elements. Show the sequence of unions that would create a tree of this depth. What does this say about the $O(1)$ amortized running time for disjoint set operations?
3. You have a width-b binary counter that you can increment, as in the example in the text.
   (a) When the counter overflows, it resets to 0. Show the amortized cost for an increment is $O(1)$.
   (b) When the counter overflows, it stays at $111\ldots1$ (that is, b ones in a row). What is the amortized cost for an increment in this situtation? How could we make the amortized cost $O(1)$?
   (c) What if we allow the decrement operation – what does that do to the amortized cost? Can we improve this?
4. (10 points) An undirected graph G can be 2-colored if every vertex in the graph can be colored either red or green, such that no two adjacent vertices have the same color. That is, for each edge (u, v), color$[u] \neq$ color$[v]$. Give an algorithm that takes as input a graph G and returns true if the graph can be 2-colored, and false otherwise. You may assume that G is a well-formed undirected graph (That is, if (u, v) is in G, then (v, u) will also be in G)