Note Title 8/12/2014

① $\varphi_{e_1} \circ \varphi_{e_2}$ IS PARTIALLY COMPUTABLE.

$\varphi_{e_1}(\varphi_{e_2}(n))$

WHAT IS THE DOMAIN OF THE COMPOSITION?

② IF φ_{f_1} AND φ_{f_2} ARE (TOTAL) COMPUTABLE FUNCTIONS, SO IS

$\varphi_{f_1} \circ \varphi_{f_2}$.

③ IF φ_{e_1} AND φ_{e_2} ARE PCFs, THEN SO IS $\varphi_{e_1} + \varphi_{e_2}$. WHAT'S THE DOMAIN?

④ LET $f(n) = \begin{cases} 0 & \text{IF } n \leq 2 \\ p & p \text{ is greatest prime} < n, \text{ IF } n > 2. \end{cases}$

IF $g(x) = \begin{cases} 1 & \text{IF } x \text{ IS PRIME} \\ 0 & \text{IF } x \text{ IS NOT PRIME.} \end{cases}$ IS C.F., SO IS f.

Picture of $\varphi_{e,s}(n)$.

n (inputs) ↓ \ steps s →	0	1	2	3	4	$\cdots$		s $\cdots$
0	-1	-1	-1	-1	3	3	3	3 $\cdots$
1	-1	0	0	0	0	0	0	0 $\cdots$
2	-1	-1	-1	-1	-1	-1	17	17 $\cdots$
3	-1	-1	-1	-1	-1	-1	$\cdots$	
4								
$\vdots$								
n								$\boxed{\varphi_{e,s}(n)}$
$\vdots$								

THM $A^{\neq\emptyset}$ IS C.E. IFF A IS RANGE OF A (TOTAL) COMPUTABLE FUNCTION.

Pf $\longrightarrow$ A IS A C.E. SET, SO A IS DOMAIN OF φ_e FOR SOME e.

NEED TO BUILD COMP. FUNCTION, f, S.T. RANGE OF f IS A. FIX $a_0 \in A$.

$$f \begin{cases} \text{INPUT } x = \langle n, s\rangle \\ \text{IF } \varphi_{e,s}(n) \neq -1, \text{ OUTPUT } n \\ \text{ELSE OUTPUT } a_0. \end{cases}$$

CLAIM: $\text{rng } f = A$.

1. SUPPOSE $a \in A$. THEN $\varphi_e(a)\downarrow$ IN s_a STEPS FOR SOME s_a, SO $f(\langle a, s_a\rangle) = a \in \text{rng } f$.

2. SUPPOSE $n \in \text{rng}$ of f. THEN $n = a_0 \in A$ OR $\varphi_{e,s}(n) \neq -1$, FOR SOME s, SO $n \in A$.

$\longleftarrow$ (IF A IS rng of COMPUTABLE FUNCTION g, THEN A IS C.E. (DOMAIN OF SOME P.C.F.))

WE NEED TO WRITE A PROG. THAT ONLY HALTS ON INPUT n IF $n \in A$.

INPUT n

SET $x = 0$.

Q: WHAT HAPPENS IF $n \notin \text{rng } g$? CRASH

$n \in \text{rng } g$? HALT.

1. $g(x) = n$? YES, OUTPUT 0.

NO, $x = x+1$, GOTO 1.

$\blacksquare$

EXERCISE $h(j,k)$ COMP. FCTN. DEFINE $f_j(k) = h(j,k)$

$f_0(k)$: $f_0(0) = h(0,0)$, $f_0(1) = h(0,1)$, ...

$f_1(k)$: $f_1(0) = h(1,0)$, $f_1(1) = h(1,1)$, ...

$\vdots$

$f_j(k)$: $f_j(0) = h(j,0)$, $f_j(1) = h(j,1)$.

$\vdots$

FIND $A \subseteq \mathbb{N}$

S.T. $\forall j$

$\chi_A \neq f_j$