PROJECT: MEMORY LIMITATION

ALGORITHMIC LEARNING THEORY, SUMMER 2014

1. Introduction

Do you remember the first sentence you ever heard? Do you remember all the sentences you've heard today? Of course not. In our general identification paradigm, there are no memory constraints built in to our model: Our learner is allowed to review *every* member of the text before offering a new conjecture. What happens if we disallow learners with perfect memories? In this project, you'll explore how *memory limited* learning affects the classes of languages that can be learned.

2. Computability theory: Lemmas and exercises

Exercise 1. *Let $D = \{d_0 < d_1 < \ldots < d_k\}$ be a finite set of natural numbers. Define $i_D = \sum_{j=0}^{k} 2^{d_j}$. Find a computable function h so that $h(i_D)$ gives an index for the set D (i.e., $W_{h(i_D)} = D$).*

Exercise 2. *Let $L = \{\langle 0, x\rangle \mid x \in \mathbb{N}\}$, and for each $i \in \mathbb{N}$, let*

$$\begin{aligned} L_i &= \{\langle 0, x\rangle \mid x \in \mathbb{N}\} \cup \{\langle 1, i\rangle\} \\ L'_i &= \{\langle 0, x\rangle \mid x \neq i\} \cup \{\langle 1, i\rangle\}. \end{aligned}$$

Find a computable function h so that

$$h(x) = \begin{cases} \text{an index for } L, & x = 0 \\ \text{an index for } L_i, & x = 2i+1 \\ \text{an index for } L'_i, & x = 2i+2 \end{cases}$$

3. Learning Theory I: Identification, lemmas and exercises

Proposition 1. *Let $\mathcal{L}$ be the collection of languages defined in Exercise 2. Then $\mathcal{L}$ is identifiable. Moreover, $\mathcal{L}$ is recursively identifiable.*

4. Learning Theory II: Limitations

Definition 1. *For all non-empty $\sigma \in SEQ$, define*

$$\begin{aligned} \sigma^- &= \sigma \text{ with the last member deleted, and} \\ \sigma^- n &= \text{ only the last } n \text{ members of } \sigma. \end{aligned}$$

So, for example, if $\sigma = (2, 67, 3, 10, 11, 31)$, then $\sigma^- = (2, 67, 3, 10, 11)$ and $\sigma^- 3 = (10, 11, 31)$.

Definition 2. *Let $\varphi \in \mathcal{F}$ be a learning function. We say φ is $n-$*memory limitied *if for all σ and τ in SEQ, if $\sigma^- n = \tau^- n$ and $\varphi(\sigma^-) = \varphi(\tau^-)$, then $\varphi(\sigma) = \varphi(\tau)$.*

Proposition 2. *RE_{FIN} is identifiable by a recursive, (1-)memory limited learner.*

Proposition 3. $[\mathcal{F}^{ML}] \subsetneq [\mathcal{F}^{REC}]$.