DC FELLOWS PROGRAM
MATH IMMERSION SUMMER COURSE

ALGEBRA AND NUMBER THEORY PRACTICE
JUNE 2, 2008

1. Multiple choice problems

(1) Simplify: $6\sqrt{192} + 5\sqrt{12}$

(a) $2967\sqrt{7125}$
(b) $\sqrt{204}$
(c) $58\sqrt{6}$
(d) $1682\sqrt{6}$
(e) $841\sqrt{12}$
(f) $58\sqrt{3}$

(2) The equation $a + (k + c) = k + (c + a)$ is an illustration of which of the following properties?

(a) Distributivity and commutativity.
(b) Commutativity and associativity.
(c) Associativity and distributivity.
(d) Associativity and inversion.

(3) What is the least common multiple (LCM) of 27, 90, and 84.

(a) 90
(b) 3780
(c) 204120
(d) 1260
(e) 3

(4) Solve for x.

$$2x^2 - 3x - 2 = 0$$

(a) $x = \{1, 2\}$
(b) $x = \{0.5, -2\}$
(c) $x = \{-0.5, 2\}$
(d) $x = \{1, -2\}$

(5) What would be the total cost of a suite for \$295.99 and a pair of shoes for \$69.95 including the 6.5% sales tax?

(a) \$389.73
(b) \$398.37
(c) \$237.86
(d) \$315.23

(6) Solve for x.

$$17 > 4 + |2x - 1|$$

(a) $(-7, 7)$
(b) $(-6, 7)$
(c) $(-\infty, -7) \cup (6, \infty)$
(d) There is no solution.

(7) Which of the following is *false*.

(a) $(x^2y^3)^2 = x^4y^6$
(b) $m^2(2n)^3 = 8m^2n^3$
(c) $\frac{m^3n^4}{m^2n^2} = mn^2$
(d) $(x + y^2)^2 = x^2 + y^4$

(8) Which of the following divides $9x^2 + 6x - 35$?

(a) $x + 5$
(b) $3x - 7$
(c) $x + 7$
(d) $3x - 5$

(9) Which of the following is a factor of $18 + 144m^3$

(a) $1 + 2m$
(b) $1 - 8m$
(c) $1 + m - 2m$
(d) $1 - m + 2m$

(10) Use the examples below to determine the value of $5 \circledast 4$.

$$4 \circledast 3 = 13 \qquad 7 \circledast 2 = 47$$
$$3 \circledast 1 = 9 \qquad 1 \circledast 5 = -4$$

(a) 20
(b) 29
(c) 1
(d) 21

(11) What is the smallest number that is divisible by 3 and 5 and leaves a remainder of 3 when divided by 7?

(a) 15
(b) 18
(c) 25
(d) 45

(12) Which of the following is necessarily composite whenever x is odd, y is even, and both are greater than or equal to 2?

(a) $x + y$
(b) $3x + 2y$
(c) $5xy$
(d) $5x + 3y$

(13) What is the units digit of 33^{1000}?

(a) 1

(b) 3
(c) 7
(d) 9

(14) Which of the following is not closed under addition?

(a) Natural numbers (without zero)
(b) Integers
(c) Positive, rational numbers greater than 1
(d) Odd numbers

(15) Which of the following is not a group under addition?

(a) Natural numbers
(b) Integers
(c) Rational numbers
(d) Real numbers

(16) In the course of solving an equation, which of the following procedures can result in an equation that yields a real root that does not satisfy the original equation (i.e. introduces an extraneous solution)?

(a) Subtracting the same number from both sides of the equation.
(b) Raising both sides of the equation to the third power.
(c) Squaring both sides of the equation.
(d) Dividing both sides of the equation by a nonzero number.

2. Structured response problems

(1) (a) Find all of the factorizations of 500 into pairs of natural numbers.
   (b) Which pair has the highest GCF?
   (c) State the Fundamental Theorem of Arithmetic.

(2) A town has been experiencing a boom in population growth. In the year 2000, the population was 45,000, and by 2005, it had grown to 60,000.
   (a) Assuming a linear relationship, find the average annual rate of change of the population.
   (b) Using t to represent years, give an algebraic expression representing the population of the town t years after 2000.
   (c) Use this expression to predict the population of the town in the year 2010.

(3) A florist sells two special floral arrangements, *regular* and *deluxe*. These arrangements use three varieties of fresh flowers – carnations, roses, and tulips. The regular arrangement requires 2 carnations, 1 rose, and 2 tulips. The deluxe arrangements each contain 4 carnations, 2 roses, and 3 tulips.

   The arrangements are prepared at the beginning of the day and then stored under refrigeration for later sales to customers. For a certain holiday, the florist will have available 160 carnations, 90 roses, and 140 tulips.

   The florist will make a profit of \$5.50 on each regular arrangement and \$7.50 on each deluxe arrangement that she sells. Assuming she sells everything she makes, how many of each type of arrangement should she make in order to maximize her profits? What will that maximum profit be? *Show your reasoning.*

(4) Show that the cube root of 3 is irrational.