

DC FELLOWS PROGRAM
MATH IMMERSION SUMMER COURSE

CALCULUS, CONTINUED
JUNE 9, 2008

- (1) Refer to the graph below. The area of region I is 4. The area of region II is 6, and the area of region III is 1. Find

(a) $\int_{-1}^0 f(x) \, dx$

(b) $\int_{-1}^3 -2f(x) \, dx$

(c) $\int_2^{-1} \frac{f(x)}{\pi} \, dx$

- (2) Compute the following indefinite integrals:

(a) $\int x^4 + \sqrt{x} + \frac{1}{x^2} - \cos x \, dx$

(b) $\int \cos x \, e^{\sin x} \, dx$

(c) $\int x(4x^2 - 1)^{18} \, dx$

- (3) Compute the following definite integrals:

(a) $\int_1^3 t^2 - 1 \, dt$

(b) $\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \cos 2x \, dx$

(c) $\int_0^1 xe^{-x^2} \, dx$

- (4) If $F(x) = \int_0^{\sin x} \ln(4t^2 - t) \, dt$, find $F'(x)$.

- (5) Solve the initial value problem: $\frac{dy}{dx} = 3x^2 - \sqrt{x}$, and when $x = 1$, $y = 0$.

- (6) Find the area of the region bound by $y = 3x - 2$ and $y = -x^2 + 4$.

- (7) Convert $0.\overline{123}$ to fraction form. Show your work, and try it on your calculator.

- (8) Write a general term for the following sequence:

$$1, \frac{4}{3}, \frac{9}{5}, \frac{16}{7}, \frac{25}{9}, \dots$$