DC FELLOWS PROGRAM
MATH IMMERSION SUMMER COURSE

CALCULUS, JUNE 5, 2008

1. Multiple choice problems

(1) If $\lim_{x\to c} f(x) = 0$ and $\lim_{x\to c} g(x) = 0$, what can be concluded about $\lim_{x\to c} \dfrac{f(x)}{g(x)}$?

(a) The value is not finite.
(b) The value is 0.
(c) The value is 1.
(d) The value cannot be determined from the information given.

(2) If $y = e^2$ then $y' = 2e$.
(a) True
(b) False

(3) $\frac{d}{dx}(10^x) = x10^{x-1}$
(a) True
(b) False

(4) $\frac{d}{dx}(\ln(72)) = \frac{1}{72}$.
(a) True
(b) False

(5) If $f(x)$ is a differentiable function, then the derivative of $\sqrt{f(x)}$ is $\frac{f'(x)}{2\sqrt{f(x)}}$.
(a) True
(b) False

(6) If $g(x) = x^3$, then $\lim_{h\to 0} \frac{g(2+h)-g(2)}{h} = 12$.
(a) True
(b) False

(7) Which of the following is a **TRUE** statement:
(a) If a function is continuous at $x = a$, then it is differentiable there.
(b) If a function is differentiable at $x = a$, then it is continuous there.

(8) In a certain chemical reaction, the number of grams, N, of a substance produced t hours after the reaction begins is given by $N(t) = 16t - 4t^2$, where $0 < t < 2$. At what rate, in grams per hour, is the substance being produced 30 minutes after the reaction begins?

(a) 7
(b) 12
(c) 16
(d) 20

(9) L'Hôpitàl's rule provides a method for evaluating which of the following?
  (a) The derivative of a function.
  (b) The limit of a function.
  (c) The sum of an arithmetic series.
  (d) The sum of a geometric series.

(10) If $f'(a) = 0$ and $f''(a) < 0$ then $f(x)$
  (a) has a discontinuity at $x = a$.
  (b) has a point of inflection at $x = a$.
  (c) has a local maximum at $x = a$.
  (d) has a local minimum at $x = a$.
  (e) is decreasing at $x = a$.

(11) If $f'(c) = 0$ then $f(x)$ has a local minimum or a local maximum at $x = c$.
  (a) True
  (b) False

(12) If $f(x)$ has an local minimum at $x = c$, and $f'(c)$ exists, then $f'(c) = 0$.
  (a) True
  (b) False

(13) If $f'(x)$ exists and is non-zero for all values of x, then $f(0) \neq f(1)$. THINK!
  (a) True
  (b) False

(14) There is a function, $f(x)$, so that $f(0) = 0$ and $f(1) = 3$ and $f'(x) > 3$ for all values of x.
  (a) True
  (b) False

(15) Which of the following statements is a **TRUE** statement:
  (a) If f is continuous at a point, it is differentiable there.
  (b) If f is differentiable at a point, it is continuous there.

(16) If $f(x) = sin(5)$ then $f'(x) = cos(5)$.
  (a) True
  (b) False

(17) Find the area under the function $y = x^2 + 4$ from $x = 3$ to $x = 6$.
  (a) 75
  (b) 21
  (c) 96
  (d) 57

(18) If the velocity of a body is given by $v = 16 - t^2$, find the distance travelled from $t = 0$ until the body comes to a complete stop.
  (a) 16
  (b) 43
  (c) 48

(d) 64

(19) Find the following limit: $\lim_{x \to 2} \frac{x^2 - 4}{2 - x}$

(a) 0
(b) -2
(c) -4
(d) Does not exist.

(20) Find the first derivative of the function $f(x) = (\sin x + 4)(3x^2 - 7x)^5$.

(a) $f'(x) = (\cos x)(5(6x - 7)^4)$
(b) $f'(x) = (\cos x + 4)(5(3x^2 - 7x)^4(6x - 7))$
(c) $f'(x) = (\cos x)(3x^2 - 7x)^5 + (\sin x + 4)(5(3x^2 - 7x)^4(6x - 7))$
(d) $f'(x) = (\cos x)(3x^2 - 7x)^5 + (\sin x + 4)(6x - 7)^4$

(21) Find the absolute maximum value attained by the function $y = 2x^2 + 3x$ on the interval from $x = 0$ to $x = 3$.

(a) -3/4
(b) -4/3
(c) 0
(d) 27

(22) Find the indefinite integral:

$$\int 4x^3 + \frac{6x - 2x^2}{x}\ dx$$

(a) $x^4 - x^2 + 6x + C$
(b) $x^4 - \frac{2}{3}x^3 + 6x + C$
(c) $12x^2 - 2 + C$
(d) $\frac{4}{3}x^4 - x^2 + 6x + C$

(23) Which of the following is an antiderivative of e^{3x}?

(a) $3xe^{3x}$
(b) $3e^{3x} + 2$
(c) $\frac{1}{3}e^{3x} - \pi$
(d) $\frac{1}{3}e^{x} + 2$

(24) Evaluate: $\int_0^2 x^2 + x - 1\ dx$.

(a) 11/3
(b) 8/3
(c) -8/3
(d) -11/3

(25) Find the limit.

$$\lim_{x \to 0} \frac{\sin 2x}{5x}$$

(a) ∞
(b) 0
(c) 1.4
(d) 1

(26) Find the sum: $\displaystyle\sum_{n=2}^{\infty} \frac{3}{4^n}$

(a) 0.25
(b) 1
(c) 4
(d) This series does not converge to a finite sum.