DC FELLOWS PROGRAM
MATH IMMERSION SUMMER COURSE

DISCRETE MATHEMATICS
JUNE 11, 2008

1. Multiple choice problems

(1) Given the recursive function defined by

$$f(0) = 3, \text{ and}$$
$$f(n) = 2f(n-1) + 3 \text{ for } n \geq 1,$$

what is the value of $f(3)$?

(a) 9
(b) 21
(c) 45
(d) 93

(2) How many different license plates can be made consisting of two digits followed by four letters?

(a) $2 \cdot 4$
(b) $({}_{10}C_5)({}_{26}C_4)$
(c) $10^2 \cdot 26^4$
(d) 36^6

(3) A civic club has 200 members. A committee of 5 members is to be selected to attend a national conference. How many different committees could be formed?

(a) ${}_{200}C_5$
(b) ${}_{200}P_5$
(c) 200^5
(d) 200!

(4) How many different license plates having 3 distinct letters followed by 4 numbers can be made?

(a) $26^3 \cdot 10^4$
(b) $({}_{26}C_3)({}_{10}C_4)$
(c) $({}_{26}P_3)({}_{10}C_4)$
(d) $({}_{26}P_3)(10^4)$

(5) Given the sequence defined recursively by

$$a_1 = -3, \text{ and}$$
$$a_n = a_{n-1} - 6,$$

What is a_4?

(a) -2
(b) -9
(c) -10
(d) -21

(6) Which of the following is not a tree?

(a)

(b)

(c)

(d)

(7) Which of the following is a false statement.

(a) In a graph, the degrees of the vertices and the number of vertices cannot both be odd.
(b) A connected graph having 3 vertices must have at least two edges.
(c) A connected graph having 4 vertices must have at least 3 edges.
(d) The sum of the degrees of the vertices in a tree can be odd.

(8) Which of the following is not arithmetic sequence?

(a) $5, 5, 5, 5, 5, \ldots$
(b) $1, 1/3, 1/9, 1/27, 1/81, \ldots$
(c) $7, 7/2, 7/4, 7/16, \ldots$
(d) $0.5, 0.3, 0.1, -0.1, \ldots$

2. Exercises

(1) Find the probability of being dealt 10 Clubs in a 13 card hand out of a standard deck. (A standard deck has 52 cars, 13 each of Clubs, Diamonds, Hearts, and Spades.)

(2) Consider the sequence $7, 13, 19, 25, 31, \ldots$.
   (a) Is this sequence arithmetic, geometric, or neither?
   (b) Write down the recursive definition for this sequence.
   (c) Write down the explicit definition of the sequence.
   (d) Find the sum of the first 37 terms of this sequence.

(3) Consider the sequence $4, 12, 36, 108, \ldots$.
   (a) Is this sequence arithmetic, geometric, or neither?
   (b) Write down the recursive definition for this sequence.
   (c) Write down the explicit definition of the sequence.
   (d) Find the sum of the first 37 terms of this sequence.

(4) Does the graph below have an Euler circuit? If it does, find one. Does it have a Hamiltonian path? If it does, find one.

(5) Draw a graph having 5 vertices and at least two edges that is connected. Now draw one that is not connected. Now draw one that is a tree.