DC FELLOWS PROGRAM
MATH IMMERSION SUMMER COURSE

FUNCTIONS
JUNE 4, 2008

1. EXERCISES

(1) Find the domain and range:
  - $h(x) = \frac{1}{x}$
  - $y = \sqrt[6]{5x^2 - 1}$
  - $f(x) = \frac{1}{\sqrt{3x-2}}$
  - $g(t) = \sec x$

(2) Given that $p(x) = 2x^2 - \sqrt{x}$, find $p(5r - 4) + 2p(r^3)$.

(3) Determine whether the functions below are even, odd, or neither.
  - $y = \sqrt[3]{2x^5 - x}$
  - $y = 5x + 2$
  - $f(t) = 7^{-t}$
  - $g(x) = \sqrt{x}$
  - $f(\theta) = 4\cos(2\theta) + 1$
  - $g(\alpha) = \tan\alpha$

(4) Graph the following by shifting, scaling, and/or reflecting a basic function.
  - $y = (x - 4)^7 + 3$
  - $f(t) = 3\sin(\frac{t}{2\pi}) - 2$
  - $g(r) = \ln(2r - 1)$

(5) For data storage services, a provider charges a flat rate of $1000 for the first terabyte of data, and $0.25 for each additional gigabyte. Express the cost, y, of storage as a function of x, the number of terabytes of data to be stored on the server. (Note that a terabyte is 1000 gigabytes.) Make a graph of your function.

(6) Consider the function $f(x) = \sqrt{4 - x^2}$.
  (a) What is the largest interval on which $f(x)$ is real-valued?
  (b) What is the largest interval containing -1 for which $f^{-1}(x)$ exists?
  (c) Give a formula for the inverse function for f restricted to the interval determined in (b).

(7) Solve for x: $81^{x-2} = 9 \cdot 3^{4x}$.

(8) Solve for x: $8^x = 5^{2x^2-1}$.

(9) Given that $f(4) = 9$, $g(4) = 6$, $f(6) = 2$, and $g(9) = 13$, find
- $(f+g)(4)$
- $(f/g)(4)$
- $f(g(4))$
- $g(f(4))$

(10) Find functions f and g so that $h(x) = f(g(x))$, where $h(x) = 5\sin(3x^2 - 7)$.