

DC FELLOWS PROGRAM
MATH IMMERSION SUMMER COURSE

FUNCTIONS
JUNE 3, 2008

1. MULTIPLE CHOICE PROBLEMS

- (1) Solve for v_0 : $d = at(v_t - v_0)$.
- (a) $v_0 = atd - v_t$
 - (b) $v_0 = d - atv_t$
 - (c) $v_0 = atv_t - d$
 - (d) $v_0 = \frac{atv_t - d}{at}$
- (2) Given $f(x) = \frac{3x-2}{x}$ and $g(x) = x^2$, determine $g(f(x))$.
- (a) $\frac{3x^2-2}{x^2}$
 - (b) $\frac{9x^2+4}{3x-2}$
 - (c) $\frac{9x^2-12x+4}{x^2}$
 - (d) $3x^3 - 2$
- (3) Find the x -coordinate of the point of intersection of the functions $y = 6(x+2) - 5$ and $y = 21 - x$.
- (a) 3
 - (b) 2
 - (c) 5
 - (d) 0
- (4) How does the function $y = x^3 + x^2 + 4$ behave on the interval from $x = 1$ to $x = 3$?
- (a) It is increasing, then decreasing.
 - (b) It is increasing.
 - (c) It is decreasing.
 - (d) It is decreasing, then increasing.
- (5) The zeros of $f(x) = x^3 + x^2 - 14x - 24$ are
- (a) 4, 3, and 2.
 - (b) 3, and -8.
 - (c) 7, -2, and -1.
 - (d) 4, -3, and -2.

(6) Which of the following is equivalent to the statement $\log_x k = m$?

- (a) $x^m = k$
- (b) $k^m = x$
- (c) $x^k = m$
- (d) $m^x = k$

(7) Solve for x : $10^{x-3} + 5 = 105$.

- (a) $x = 3$
- (b) $x = 10$
- (c) $x = 2$
- (d) $x = 5$

(8) Which of the following represents a function?

I. $y^2 = 5 - x^2$

II. $\{(0, 1), (6, -2), (5, 0), (8, 0)\}$

III. $x - y = 5$

- (a) I and III only.
- (b) III only.
- (c) II and III only.
- (d) I, II, and III.

(9) Give the function $f(x) = x^3$, which best describes the graph of the function $g(x) = (x - 2)^3$?

- (a) It's the same as the graph of $f(x)$, but shifted down 2 units.
- (b) It's the same as the graph of $f(x)$, but shifted up 2 units.
- (c) It's the same as the graph of $f(x)$, but shifted left 2 units.
- (d) It's the same as the graph of $f(x)$, but shifted right 2 units.

(10) At a certain elementary school, there were 25 children in the band in 2000, and band participation has been increasing by an average of 3 students per year since then. At another school, there were 18 students in the band in 2002, and participation has increased by an average of 4 students per year since then. Which of the following equations could be used to determine when the schools will have the same size bands, assuming recent trends continue?

- (a) $25 + 3t = 18 + 4t$
- (b) $18 + 4t = 31 + 3t$
- (c) $31 = 18 + 4t$
- (d) It cannot be determined from the information given.

(11) At how many points in the xy -plane do the graphs of $y = 4x^5 - 3x^2 - 1$ and $y = -0.4 - 0.11x$ intersect?

- (a) One
- (b) Two
- (c) Three
- (d) Four

- (12) If
- (a) the graph of the function $f(x)$ is the line with slope 2 and y -intercept 1, and
 - (b) the graph of the function $g(x)$ is the line with slope -2 and y -intercept -1,
- which of the following is an algebraic representation of $y = f(g(x))$?

- (a) $y = 0$
- (b) $y = -4x - 3$
- (c) $y = -4x - 1$
- (d) $y = -(2x + 1)^2$

- (13) At the beginning of 1990, the population of rabbits in a wooded area was 250. The function

$$P(t) = 250 \cdot (3.04)^{\frac{t}{1.98}}$$

was used to model the approximate population P of rabbits in the area t years after January 1, 1990. According to this model, which of the following best describes how the rabbit population changed in the area?

- (a) The population doubled every four months.
- (b) The population tripled every 6 months.
- (c) The population doubled every 36 months.
- (d) The population tripled every 24 months.

- (14) If $f(x) = 3x^2$, what are all real values of a and b for which the graph of $g(x) = ax^2 + b$ is below the graph of $f(x)$ for all values of x ?
- (a) $a \geq 4$ and b is positive.
 - (b) $a \leq 3$ and b is negative.
 - (c) a is negative and b is positive.
 - (d) a is any real number and b is negative.

2. STRUCTURED RESPONSE PROBLEMS

- (1) A Florida farmer owns an orange grove of 100 trees, with an average yield of 70 oranges per tree. He estimates that the average yield per tree will decrease by two for each additional tree he plants.
- (a) What is the farmers current total yield?
 - (b) If he plants 3 additional trees, by how much will his total yield change?
 - (c) Express the *total* yield as a function of the number of additional trees planted.
 - (d) How many trees should he plant in total to maximize his total yield?
- (2) Consider the function f defined for all real numbers by

$$f(x) = \frac{x^4}{2} - 3x^3 - x^2 + 15x + c,$$

where c is a real-valued constant.

- (a) Determine a value of c for which the function has exactly 4 real roots.
- (b) Determine a value of c for which the function has no real roots.
- (c) Determine a value of c for which the function has exactly 2 real roots.

Explain how you arrive at your answers.