DC FELLOWS PROGRAM
MATH IMMERSION SUMMER COURSE

GEOMETRY AND TRIGONOMETRY
JUNE 3, 2008

1. Multiple choice problems

(1) Which of the following is *true*?

(a) The median and altitude of a right triangle are always the same segment.
(b) The median and altitude of a triangle may be different segments.
(c) The median and altitude of an isosceles triangle are always the same segment.
(d) The median and altitude of a triangle are always different segments.

(2) Which of the following is the distance from the point $(-2, 7)$ to the line $x = 5$?

(a) -9
(b) -7
(c) 5
(d) 7

(3) Which of the following statements about a trapezoid is *false*?

(a) The sum of its interior angles is 2π radians.
(b) At least two of its interior angles must be equal.
(c) At least two of its interior angles must form a supplementary pair.
(d) All trapezoids have an axis of symmetry.

(4) If the midpoint of $(-4, y)$ and $(2, -3)$ is $(x, 1)$, what are the values of x and y?

(a) $x = -1, y = 5$
(b) $x = 3, y = 2$
(c) $x = 5, y = -1$
(d) $x = -1, y = -1$

(5) Which equation represents a circle with a diameter whose endpoints are $(0, 7)$ and $(0, 3)$?

(a) $x^2 + y^2 + 21 = 0$
(b) $x^2 + y^2 - 10y + 21 = 0$
(c) $x^2 + y^2 - 10y + 9 = 0$
(d) $x^2 + y^2 - 10y - 9 = 0$

(6) What is the degree measure of an interior angle or a regular 10-sided polygon?

(a) 216
(b) 198
(c) 162
(d) 144

(7) What is the length of the major axis of the figure given by $x^2 + 9y^2 = 36$?

(a) 4
(b) 6
(c) 2
(d) 0.5

(8) Which of the following is not equal to $\sin x$?

(a) $\sqrt{1 - \cos^2 x}$
(b) $\tan x \cos x$
(c) $(\csc x)^{-1}$
(d) $\frac{1}{\sec x}$

(9) Determine the approximate rectangular coordinates of the point with polar coordinates $(5, \frac{\pi}{3})$.

(a) $(2.5\sqrt{3}, 2.5)$
(b) $(5\sqrt{3}, 5)$
(c) $(2.5, 2.5\sqrt{3})$
(d) $(5, 5\sqrt{3})$

(10) Which of the following is equivalent to $1 - \sin^2 x$?

(a) $1 - \cos^2 x$
(b) $1 + \cos^2 x$
(c) $\frac{1}{\sec x}$
(d) $\frac{1}{\sec^2 x}$

(11) For the acute angle x, $\sin x = \frac{3}{5}$. What is $\cot x$?

(a) 5/3
(b) 3/4
(c) 4/3
(d) 1

(12) Consider the rhombus with vertices $(0, 4), (0, -4), (2, 0)$, and $(-2, 0)$. For how many angles θ, where $0 < \theta \leq 2\pi$, will a rotation about the origin by angle θ map the rhombus onto itself?

(a) One
(b) Two
(c) Four
(d) Eight

(13) In $\triangle ABC$ (not shown), the length of side AB is 12, the length of side BC is 9, and the measure of angle BAC is $30°$. What is the length of side AC?

(a) 17.10
(b) 4.73

(c) 3.68
(d) It cannot be determined from the information given.

(14) In the xy-plane, an acute angle with vertex at the origin is formed by the positive x-axis and the line with equation $y = 3x$. What is the slope of the line that contains the bisector of this angle?

(a) 3
(b) 3/2
(c) $\dfrac{\sqrt{10}+1}{3}$
(d) $\dfrac{\sqrt{10}-1}{3}$

(15) Consider a circle with radius 2 and center O. Let PC be a diameter of the circle and AP a line segment tangent to the circle at point P. Let B be the point where AC intersects the circle. If AP has length 3, what is the length of BC?

(a) 1.25
(b) 2
(c) 3.2
(d) 5

2. Structured response problems

(1) Is $y = 3x - 6$ a bisector of the line segment with endpoints $(2, 4)$ and $(8, -1)$? Show your reasoning.

(2) Prove that $\cot x + \tan x = \csc x \sec x$.

(3) Show that the lines adjoining the midpoints of the adjacent sides of a quadrilateral form a parallelogram.

(4) Consider a cube with side of length 4. Show your work in answering all the following questions.
   (a) What is the length of a diagonal of one of the faces?
   (b) What is the length of a diagonal through the cube?
   (c) Let P be the point at the center of the top face of the cube. Use similar triangles (or any other method) to find the perpendicular distance from P to a diagonal through the cube.