ALGEBRA & NUMBER THEORY.

- NUMBER SYSTEMS & PROPERTIES OF OPS & RELATIONS
- ALGEBRAIC EXPRESSIONS (INCL. EXPONENT RULES ETC.)
- SOLVING EQN'S, INEQUALITIES, & SYSTEMS OF THESE.
- ALG - GEOM AND CONICS.

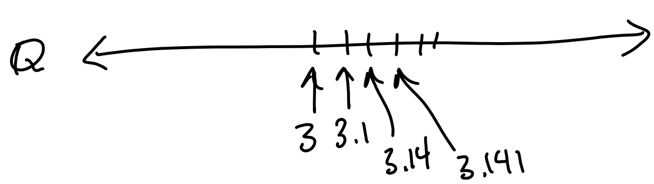
— NUMBER SYSTEMS —

- NATURAL NUMBERS, $\mathbb{N}$ $\quad 0, 1, 2, 3, 4, \ldots$
  COUNTING NUMBERS
- INTEGERS, $\mathbb{Z}$, $\quad \ldots -2, -1, 0, 1, 2, \ldots$
- RATIONAL NUMBERS — RATIOS OF INTEGERS — $\mathbb{Q}$
  $-\frac{1}{2}, 0, 1.7\overline{67}, 1.3$

$$\frac{3}{2} = 1.5 \qquad 0.9 = \frac{9}{10}$$

$$0.\overline{9} = 1 \qquad 0.\overline{34} = \frac{3}{10} + \frac{4}{100} + \ldots$$

DENSE - BETWEEN ANY TWO RATIONALS, THERE'S ANOTHER.



0.9 , 0.99
0.95

- IRRATIONAL NUMBERS — $\mathbb{P}$

  ↳ FILL THE HOLES IN RATIONALS
  NON-REPEATING, NON-TERMINATING

  CAN'T BE EXPRESSED AS THE RATIO OF INTEGERS.

- REALS — $\mathbb{R} = \mathbb{Q}$ PLUS $\mathbb{P}$

TWO KINDS OF REALS

$5 - x^3 = 0$

- ALGEBRAIC — $\sqrt[3]{5}$, $\sqrt{2}$, 7, $-\frac{3}{2}$
- TRANSCENDENTAL — NOT SOL'NS TO POLYNOMIALS

  π, e

ASIDE (BINARY)

1010.1

$2^3 \; 2^2 \; 2^1 \; 2^0 \quad 2^{-1}$

$8 + 0 + 2 + 0 + \frac{1}{2}$

$10^1 \; 10^0 \; 10^{-1}$

10.5

$0.2222\ldots$

$3^{-1} \; 3^{-2} \; 3^{-3}$

$= 1$

$x^2 + 1 = 0$ ← NO SOLUTIONS IN $\mathbb{R}$!

<u>BUT</u> HAS SOL'S IN $\mathbb{C}$.

- COMPLEX NUMBERS — $\mathbb{C}$
  - IMAGINARY NUMBERS
  - REAL NUMBERS

  CLOSED UNDER ADDITION

IMAGINARY NUMBERS

MULTS of IMAGINARY UNIT, $i = \sqrt{-1}$

$3i$, $-7i$, $1.73i$, $\frac{\pi}{2} i$

COMPLEX NUMBERS

Real Part + IMAG PART

$z = 3 + 2i$

$Re(z) = 3$
$Im(z) = 2$

ADDING COMPLEX NUMBERS –
SUBTRACTING

$(3+7i) \pm (5+6i)$

$8 + 13i$

$-2 + i$

$i = \sqrt{-1}$
$i^2 = -1$
$i^3 = i^2 \cdot i = -i$
$i^4 = (i^2)^2 = \underline{1}$

MULTIPLYING in $\mathbb{C}$.

$= -1$

$$(2-6i)\cdot(3+2i) = 6 + 4i - 18i - 12i^2$$
$$= 6 - 14i + 12 = 18 - 14i$$

DIVIDING in $\mathbb{C}$.

$$\frac{(2-6i)}{(3+2i)} \cdot \frac{(3-2i)}{(3-2i)} = \frac{-6-22i}{9+4}$$
$$= \frac{-6-22i}{13} = -\frac{6}{13} - \frac{22}{13}i$$

COMPLEX CONJUGATE OF $a+bi$ IS $a-bi$

$(a+bi)(a-bi) = a^2 - b^2 i^2$
$= a^2 + b^2$

SETS OF NUMBERS & SET OPERATIONS.

EVEN NUMBERS

$\{0, 2, 4, 6, \dots\}$ ← ORDER DOES NOT MATTER.

membership

$E = \{\underbrace{x \in \mathbb{N}}_{\text{VARIABLE \& scope}} : \underbrace{2 \text{ DIVIDES INTO } x}_{\text{Description of a property in terms of the variable.}}\}$ ← SET BUILDER NOTATION

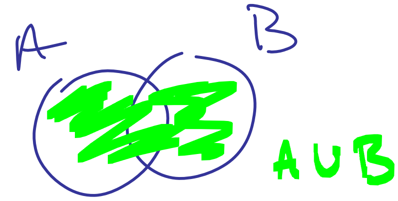
" x IS A NATURAL NUMBER AND 2 DIVIDES evenly into it."

"such that"

UNION: $A \cup B = \{x \in \mathcal{U} \mid x \in A \text{ OR } x \in B\}$

$A = \{1, 5, 7\}$ $B = \{2, 4\}$

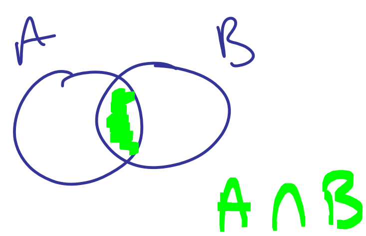
$A \cup B = \{1, 5, 7, 2, 4\}$



INTERSECTION: $A \cap B = \{x \in \mathcal{U} \mid x \in A \text{ AND } x \in B\}$

$A = \{1, 2, 3\}$ $B = \{2, 4\}$

$A \cap B = \{2\}$



COMPLEMENTATION: $\overline{A} = \{x \in \mathcal{U} \mid x \notin A\}$

$A = \{1, 2, 3\}$

$\overline{A} \stackrel{?}{=} \{4, 5, 6 \ldots\}$

$\stackrel{?}{=} \{x \in \mathbb{Z} \mid x \notin A\}$

member

$a \in S$

element

set

containment

$A \subseteq B$

sets

Relations

$=,\ <,\ \cong,\ \simeq$

$\underbrace{3 < 5}_{T}$ $\underbrace{6 \geq 8}_{F}$

- Reflexive property, $\prec$

$x \prec x$

ex IN $\mathbb{R}$

- $<$ IS NOT Reflexive

$2 \not< 2$

- $\leq$ IS REFLEXIVE

FOR ALL REALS, $x \leq x$.

Operations

$\underbrace{+,\ \cdot,\ -,\ \div}_{\text{Binary}},\ -$

negation

$3 + 2 = 5$ $\quad - -3 = 3$

COMMUTATIVE

$a + b = b + a$ ← IN $\mathbb{R}$

ASSOCIATIVE

$a + (b + c) = (a + b) + c$

DISTRIBUTIVE

$a \cdot (b + c) = a \cdot b + a \cdot c$

- Symmetric , $\prec$

IF $x \prec y$ THEN $y \prec x$

ex in $\mathbb{R}$

IF $x = y$ THEN $y = x$

So $=$ is symmetric

$<$ is not symmetric

$\leq$ is not symmetric

- Antisymmetric

IF $x \prec y$ AND $y \prec x$ THEN $x = y$

ex in $\mathbb{R}$

$<$ is not really antisymmetric (it is vacuously)

$\leq$ is legit. antisymmetric.

- Transitivity

IF $x \prec y$ AND $y \prec z$ THEN $x \prec z$.

ex in $\mathbb{R}$

$=, <$ both transitive.

$2+1 \leq 3$ AND $3 \leq 2+1$

so $3 = 2+1$

Invertable

- Has an inverse in $\mathbb{Q}$
- Does not have an inverse in $\mathbb{N}$.

ex 2 has no multiplicative inverse in $\mathbb{N}$.

2 has no additive inverse in $\mathbb{N}$.

Closure

ex Division is not closed in $\mathbb{N}$

Let $*$.

If for each choice of a, b in $\mathcal{U}$, $a * b$ is also in $\mathcal{U}$, then we say $\mathcal{U}$ is closed under $*$.

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ex

Show that $a * b \equiv a^b$ is not associative or commutative.

$2 * 3 \stackrel{?}{=} 3 * 2$

$2^3 \neq 3^2$

$2 * (3 * 2) \stackrel{?}{=} (2 * 3) * 2$

$a \circledast b = c$

$\neq$

$b \circledast a = a$

$\circledast$	a	b	c
a	b	c	c
b	a	b	a
c	b	c	a

ex

$<$	a	b	c
a	T	F	T
b	F	T	T
c	F	F	T

Reflexive?
YES.

SYMMETRIC?
NO
$b < c$ BUT $c \not< b$.

TRANSITIVE

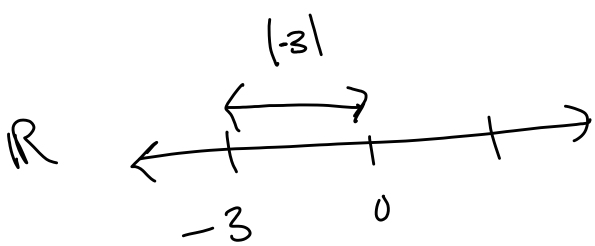
IF $x < y < z$ THEN $x < z$

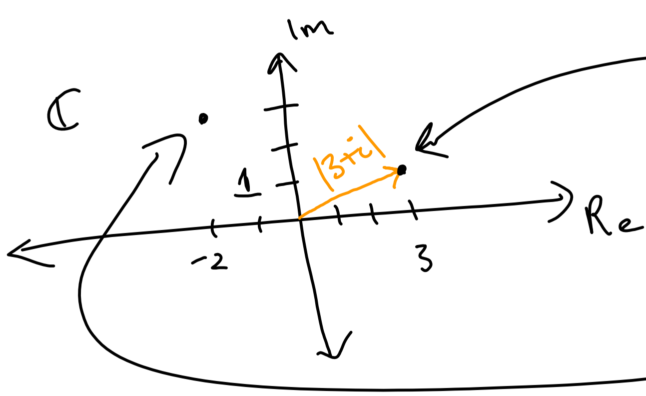
a b c

c b a

a c b

a a c





ABS VALUE OF COMPLEX.

$|3+i| = \sqrt{3^2 + 1^2} = \sqrt{10}$

$-2+3i$

ALGEBRAIC EXPRESSIONS

TERM CONSTANT OR A VARIABLE OR A PRODUCT OF THESE

ex $7, -\frac{1}{2}, \pi$ x, y, z $\pi x y z^3$

CONSTANT coeff is π

Coeff of y is $\pi x z^3$

MONOMIAL – ONE TERM

BINOMIAL – TWO TERMS

POLYNOMIALS – MULTIPLE TERMS

TERMS COMBINED w/ $+, -$.

MULTIPLYING POLYNOMIALS

$$(3x^2 - 2yz + 7)(x^4 - 1)$$

$$\frac{13}{3} = 3\overline{)13}\quad 4\,r\,1$$

POLYNOMIAL LONG DIVISION

$$\frac{3x^2 - 2x + 5}{x+1} = 3x - 5 + \frac{10}{x+1}$$

$$\begin{array}{r} 3x - 5 \\ x+1 \overline{)\,3x^2 - 2x + 5} \\ -(3x^2 + 3x) \\ \hline 0 - 5x + 5 \\ -(-5x - 5) \\ \hline 0 + 10 \end{array}$$

$\leftarrow$ Remainder

$$(x+1)(3x-5) + 10 = 3x^2 - 2x + 5$$

$$3 \cdot 4 + 1$$

EXPONENT RULES

$x^0 = 1$, BUT 0^0 UNDEFINED

$x = x^1$.

$x^a x^b = x^{a+b}$

$(x^a)^b = x^{ab}$

$\neq$

$x^{(a^b)}$

$x^a y^a = (xy)^a$

$x^{-n} = \frac{1}{x^n}$

$x^{1/n} = \sqrt[n]{x}$

$x^2 = 4$

$x = 2, -2$

But

$\sqrt{4} = 2$

EQUATION: ~~~ = ~~~

A SOL'N IS AN ASSIGNMENT OF VALUES TO VARS THAT MAKE THE STAMENT EVALUATE TO 'TRUE'

THE SOLUTION SET IS THE COLLECTION OF ALL SOL'NS.