LINES

$$y = mx + b$$
$$y - y_0 = m(x - x_0)$$
$$Ax + By = C$$

LINEAR EQNS

(x_0, y_0) IS ANY POINT ON LINE.

$Ax + By + Cz = D$

PLANE IN 3 SPACE

USE POINT SLOPE FORM TO FIND EQN OF LINE

Need SLOPE AND POINT.

$$3x - 6y = 8$$
$$y = \frac{1}{2}x - \frac{4}{3}$$

1. SLOPE: -2 (Neg recip of $\frac{1}{2}$)

   POINT: $(0, -4/3)$

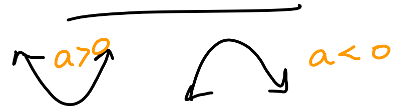
   $(y - {-4/3}) = -2(x - 0)$ $\quad y = -2x - 4/3$

2. SLOPE: $-4/3$

   POINT: $(1/3, 52/3)$

$$y = -3x^2 + 2x + 17$$
$$x_v = \frac{-2}{-3 \cdot 2} = \frac{1}{3} \; ; \; y_v = \frac{-1}{3} + \frac{2}{3} + 17$$

PARABOLAS



$$y = ax^2 + bx + c$$

$$x_v = \frac{-b}{2a}$$; For y_v PLUG IN!

$y - 52/3 = -4/3\,(x - 1/3)$

③ $8x^2 - 10x - 3 = 0$

$(2x-3)(4x+1) = 0$

$x = \frac{3}{2}$ or $x = \frac{-1}{4}$

$\sqrt[4]{x} = \sqrt[3]{x}$

$x = \left(\sqrt[3]{x}\right)^4$ $\quad (x^{1/3})^4$

$x = \sqrt[3]{x^4}$

$x^3 = x^4$

$0 = x^4 - x^3 = x^3(x-1)$

$\uparrow \quad \uparrow$

$0 \quad 1$

$7 + \sqrt[3]{2x+4} = 0$

$\sqrt[3]{2x+4} = -7$

$2x+4 = (-7)^3$

$x = \frac{-343-4}{2}$

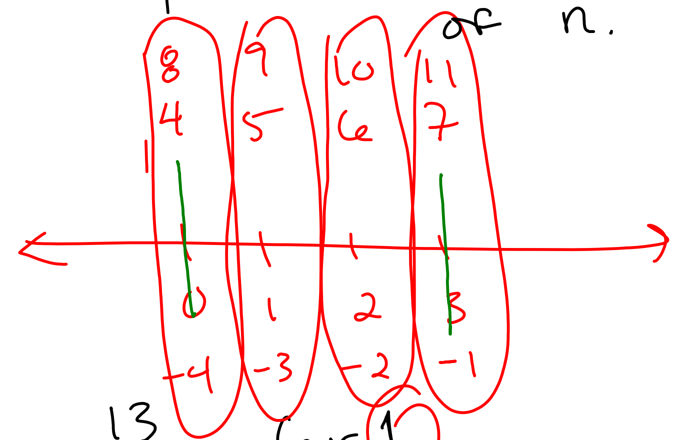
$x = -173.5$

$7\sqrt[4]{x} = 9\sqrt[6]{x}$

MODULAR ARITHMETIC

$a \equiv b \pmod{n}$ ← $|a-b|$ IS A MULTIPLE OF n.

$7 \equiv 2 \bmod 5$



(7) $\{2, 3, 6\}$

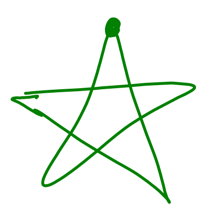
$13 \equiv 7 \bmod 2$

$13 \equiv 7 \bmod 3$

$13 \equiv 7 \bmod 6$

$\frac{13}{2} = 6 \text{ r } 1$

$\frac{7}{2} = 3 \text{ r } 1$



$\frac{360}{5} = 72°$

mod 5

i

$i^2 = -1$

$i^3 = -i$

---

(6) $p = \begin{cases} 0 & \text{IF } x < 6 \\ (x-6)^{\frac{3}{4}} + 0 & \text{IF } x \geq 6 \end{cases}$

$$p = \frac{3}{4}(x-6) + 8$$, $x \geq 6$.

$$= \frac{3}{4}(4-6) + 8$$
$$= -1.5 + 8 = 6.50$$

INEQUALITIES

$<$

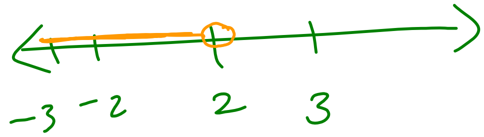
$\leq$

$+$ & $-$ Reals from both sides

$\cdot$ & $\div$ BY POSITIVE reals

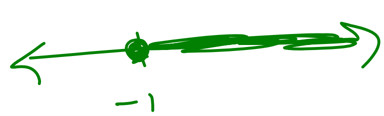
$2 < 3$

$\cdot$ & $\div$ BY NEG REAL, SWITCH SIGN.



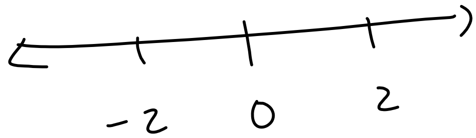
$x < 2$

$$2x - 3 \leq 7x + 2$$
$$-5x \leq 5$$
$$x \geq -1$$



ABSOLUTE VALUE

$5 + |3x - 1| = 7$



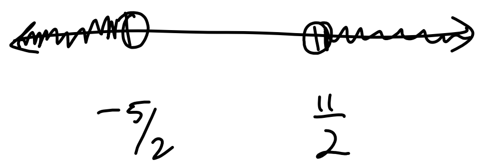
$|3x - 1| = 2$

$3x - 1 = 2$ OR $3x - 1 = -2$

$x = 1$ $\qquad$ $x = -\frac{1}{3}$

---

(8) $3 > 5 + \dfrac{|2x - 3|}{-4}$



$x > \frac{11}{2}$ OR $x < \frac{-5}{2}$

$3|2x + 11| - 1 > 14$

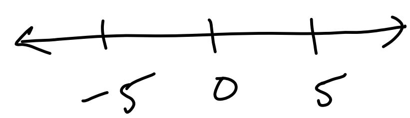
$|2x + 11| > 5$

$2x + 11 > 5$ OR $2x + 11 < -5$

⋮

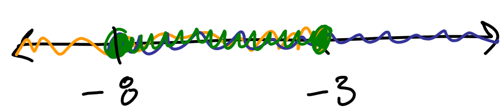
$3|2x + 11| - 1 \le 14$

$|2x + 11| \le 5$

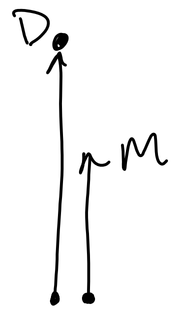


$2x + 11 \le 5$ AND $2x + 11 \ge -5$

$x \le -3$ AND $x \ge -8$



(9)



t hours after 10 am

$$D = 55t + 110$$

$$M = 85t$$

$$55t + 110 = 85t$$

$$110 = 30t$$

$$\frac{110}{30} = t$$

$$3.\overline{6} = t \text{ (hours)}$$

THEOREM IF ~~~ (ANTECEDENT / HYPOTHESIS) THEN ~~~ (CONSEQUENT / CONCLUSION).

PROOF SEQUENCE OF STATEMENTS

______ .
______ .
______ .
______ .

each is an AXIOM (POSTULATE) OR PART OF THE HYPOTHESIS OR FOLLOWS CLEARLY FROM EARLIER STATEMENTS.

THEREFORE, THM. ______

4 BASIC TYPES.

"$P \to Q$" means "P IMPLIES Q"

ASSUME OUR THM: IF P THEN Q.

① DIRECT PROOF

$$P \to S_1 \to S_2 \to \dots \to Q.$$

ex **Thm** IF $a|b$ AND $b|c$ THEN $a|c$.

PROOF $a|b$ means $ax = b$ FOR SOME $x \in \mathbb{N}$.

$b|c$ means $by = c$ FOR SOME $y \in \mathbb{N}$.

So, SUBSTITUTING IN FOR b YIELDS

$$(ax)y = c.$$

NOW WE $a \cdot xy = c$ AND xy IS A NATURAL NUMBER, SO $a|c$.

② PROOF BY CONTRADICTION. $P \to Q$

ASSUME P IS TRUE AND Q IS FALSE, AND ARRIVE AT A CONTRACTION.

THM $\sqrt{2}$ IS IRRATIONAL. (IF $\underbrace{x=\sqrt{2}}_{P}$ THEN $\underbrace{x \text{ IS IRRAT}}_{Q}$)

PROOF SUPPOSE $\underbrace{x=\sqrt{2}}_{P}$ AND $\underbrace{x \text{ IS RATIONAL.}}_{\text{NOT } Q}$

IF x IS RATIONAL THEN WE CAN WRITE

$x = \dfrac{m}{n}$, A <u>REDUCED</u> <u>RATIO</u> OF INTEGERS.

THEN $nx = m$ AND

$$n^2 x^2 = m^2, \text{ SO}$$

$$2n^2 = m^2.$$

WE SEE THAT m^2 IS EVEN, SO m

m MUST BE. WE CAN WRITE $m = 2 \cdot k$

FOR SOME NUMBER k.

WE HAVE
$$2n^2 = (2k)^2$$
$$2n^2 = 4k^2$$
$$n^2 = 2k^2$$

SO n^2 MUST ALSO BE EVEN, AND SO

n. BUT m AND n $\underbrace{\text{HAVE NO COMMON FACTORS!}}_{\text{relatively prime}}$

WE HAVE A CONTRADICTION, SO SOMETHING

WE ASSUMED MUST BE FALSE — ie. x MUST NOT BE RATIONAL.

---

(3) THM $a^2 + b^2 \geq 2ab$ FOR $a, b \in \mathbb{R}$.

proof

THE DIFFERENCE OF REALS IS REAL, AND THE SQUARE OF A REAL IS ALWAYS ≥ 0, SO FOR REALS a, b,

$$(a-b)^2 \geq 0.$$

EXPANDING, WE HAVE

$$a^2 - 2ab + b^2 \geq 0,$$ AND

SO $a^2 + b^2 \geq 2ab$.

$$a^2 - 2ab + b^2 \geq 0$$

$$(a-b)^2 \geq 0$$

---

(4) THE SUM OF TWO RATIONALS IS RATIONAL.

proof Let x AND y BE RATIONAL NUMBERS.

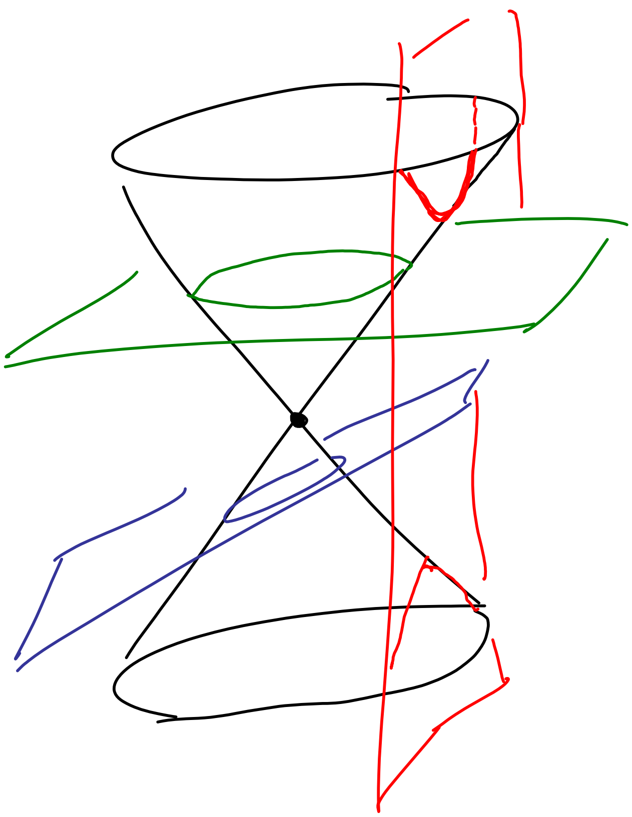
THEN $x = \frac{a}{b}$ AND $y = \frac{c}{d}$ FOR SOME

a, b, c, d in $\mathbb{Z}$, WITH $b, d \neq 0$.

COMPUTING THE SUM, WE HAVE

$$\frac{a}{b} + \frac{c}{d} = \frac{ad + bc}{bd}.$$

BOTH $(ad+bc)$ AND bd ARE INTEGERS SINCE $\mathbb{Z}$ IS CLOSED UNDER $+$ AND $\cdot$.



CONIC SECTIONS

PARABOLAS

CIRCLES

ELLIPSES

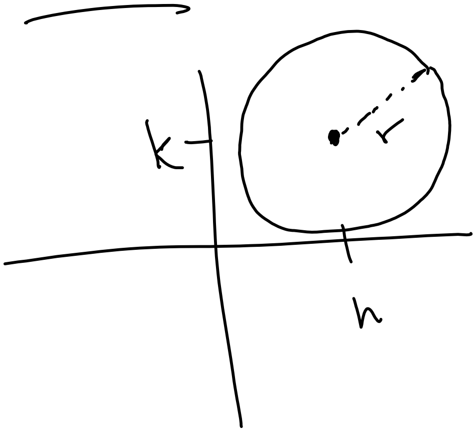
HYPERBOLAS

PARABOLAS

$$y = a(x-h)^2 + k$$

VERTEX AT (h, k)

CIRCLES



$$r = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$r^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$$

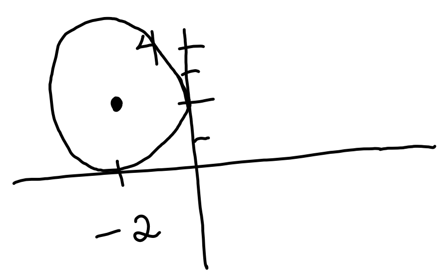
h k

$$r^2 = (x-h)^2 + (y-k)^2$$

GRAPH

$$(x-3)^2 + y^2 = 16$$

WRITE THE EQN FOR

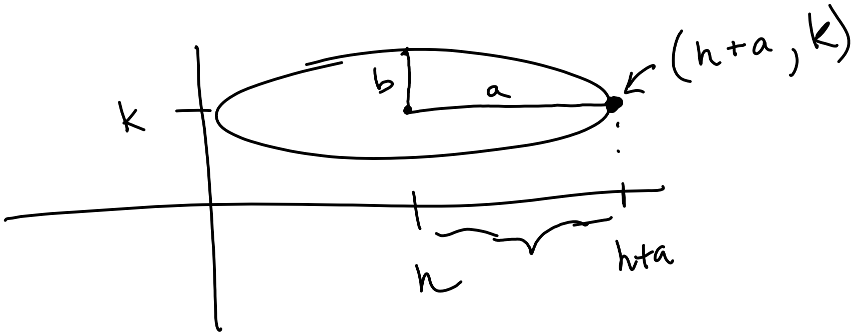


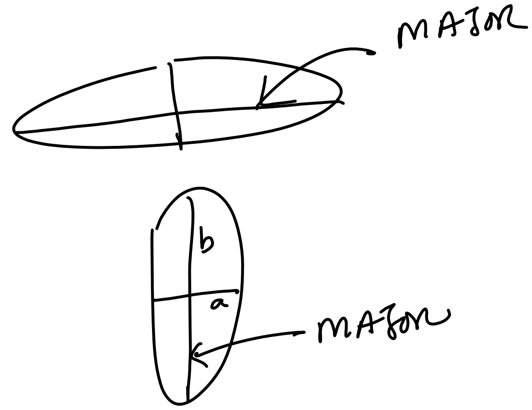
$$(h,k) = (-2, 2)$$

$$(x+2)^2 + (y-2)^2 = 4$$

ELLIPSES

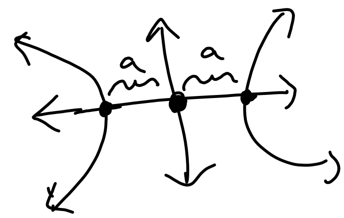
$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$$



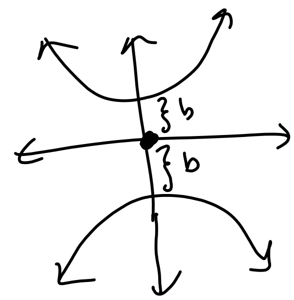


HYPERBOLAS

$$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$$



$$\frac{(y-k)^2}{b^2} - \frac{(x-h)^3}{a^2} = 1$$



COMPLETE THE SQUARE

$x^2 + 6x + 5 = 0 \longrightarrow (\ldots)^2 = \#$

$(\ldots) = \pm\sqrt{\#}$

$\frac{1}{2}$ of this is 3

ADD & SUBTRACT ITS SQUARE (9)

$$\underbrace{x^2 + 6x + 9}\; - 9 + 5 = 0$$

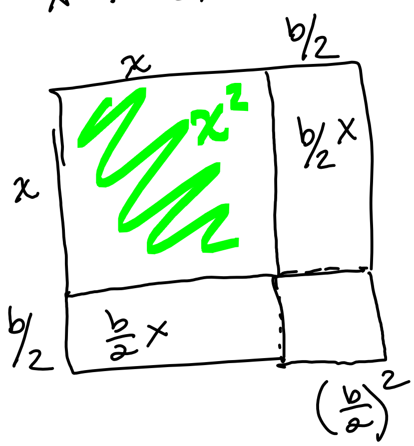
$$(x+3)^2 - 4 = 0$$

$$(x+3)^2 = 4$$

$$x+3 = \pm 2$$

$$x = -1, -5.$$

$$x^2 + bx = x^2 + \frac{b}{2}x + \frac{b}{2}x$$



$$\left(x+\frac{b}{2}\right)^2$$