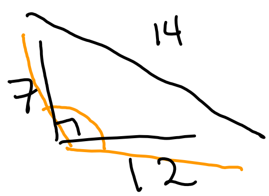
5. $7 - 12 - 14$



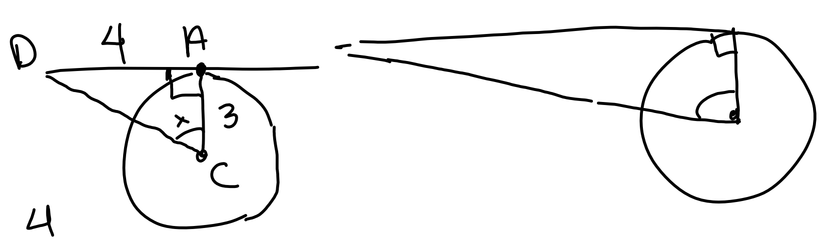
$7^2 + 12^2 < 14^2$

6.

$$\frac{45^\circ}{360^\circ} = \frac{A_{\triangle}}{A_{O}}$$

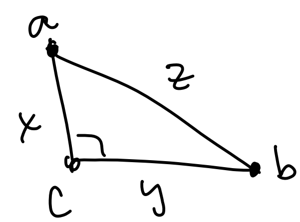
$$l = 1.5 \cdot \frac{\pi}{4} = \frac{3\pi}{8} \text{ unit}$$

7.



$\tan x = \frac{4}{3}$

PYTHAGOREAN THM



$x^2 + y^2 = z^2$

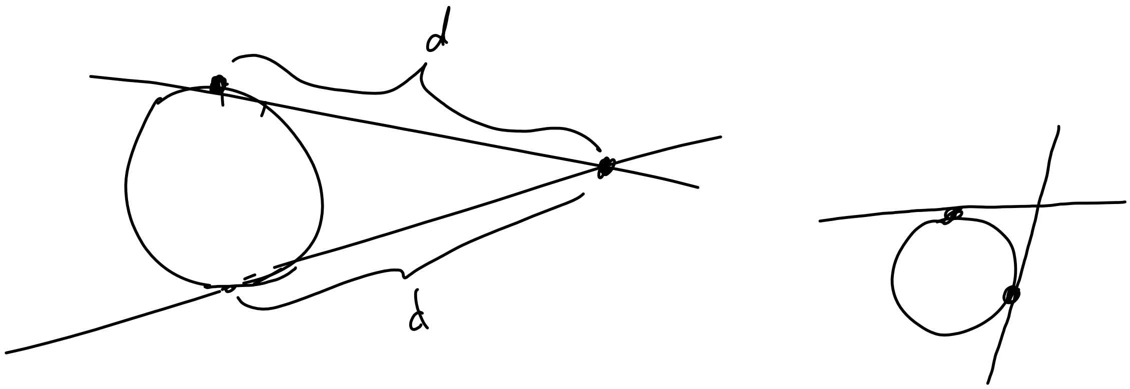
IF $\triangle abc$ IS A right $\triangle$ then $x^2 + y^2 = z^2$.

$P \to Q$ IMPLICATION

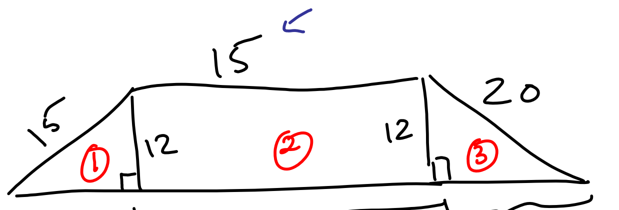
$Q \to P$ CONVERSE

IF $x^2 + y^2 = z^2$ THEN $\triangle abc$ IS RIGHT $\triangle$.

8.



9.



$$\sqrt{15^2-12^2} + 15 + \sqrt{26^2-12^2}$$

$$\left.\begin{array}{l} ① \quad \frac{1}{2}\cdot\sqrt{15^2-12^2}\cdot 12 = 54 \\ ② \quad 12\cdot 15 = 180 \\ ③ \quad \frac{1}{2}\sqrt{20^2-12^2}\cdot 12 = 96 \end{array}\right\} 330$$

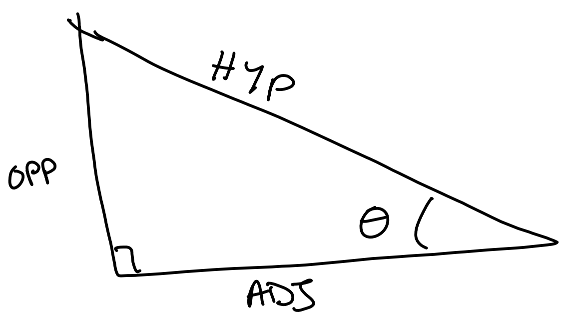
TRIGONOMETRY

RIGHT Δ's — PYTHAGOREAN'S THM

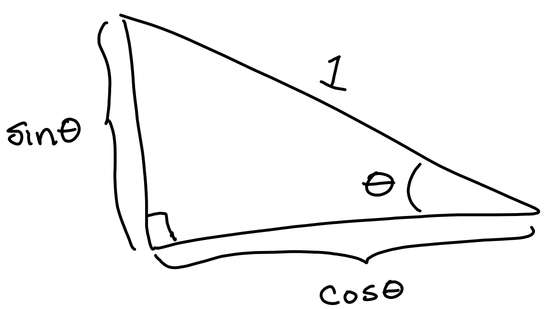
SOHCAHTOA

$$\sin\theta = \frac{OPP}{HYP}$$

$$\cos\theta = \frac{ADJ}{HYP}$$



$$\tan\theta = \frac{OPP}{ADJ} = \frac{\frac{OPP}{HYP}}{\frac{ADJ}{HYP}} = \frac{\sin\theta}{\cos\theta}$$



$$\sin^2\theta + \cos^2\theta = 1$$

$$\sec\theta = \frac{1}{\cos\theta} = \frac{HYP}{ADJ}$$

$$\csc\theta = \frac{1}{\sin\theta} = \frac{HYP}{OPP}$$

$$\cot\theta = \frac{1}{\tan\theta} = \frac{ADJ}{OPP} = \frac{\cos\theta}{\sin\theta}$$

$$1 + \frac{\cos^2\theta}{\sin^2\theta} = \frac{1}{\sin^2\theta}$$

$$1 + \cot^2\theta = \csc^2\theta$$

$$\tan^2\theta + 1 = \sec^2\theta$$

$\sin^5\theta = (\sin\theta)^5$

$\tan^2\theta = (\tan\theta)^2$

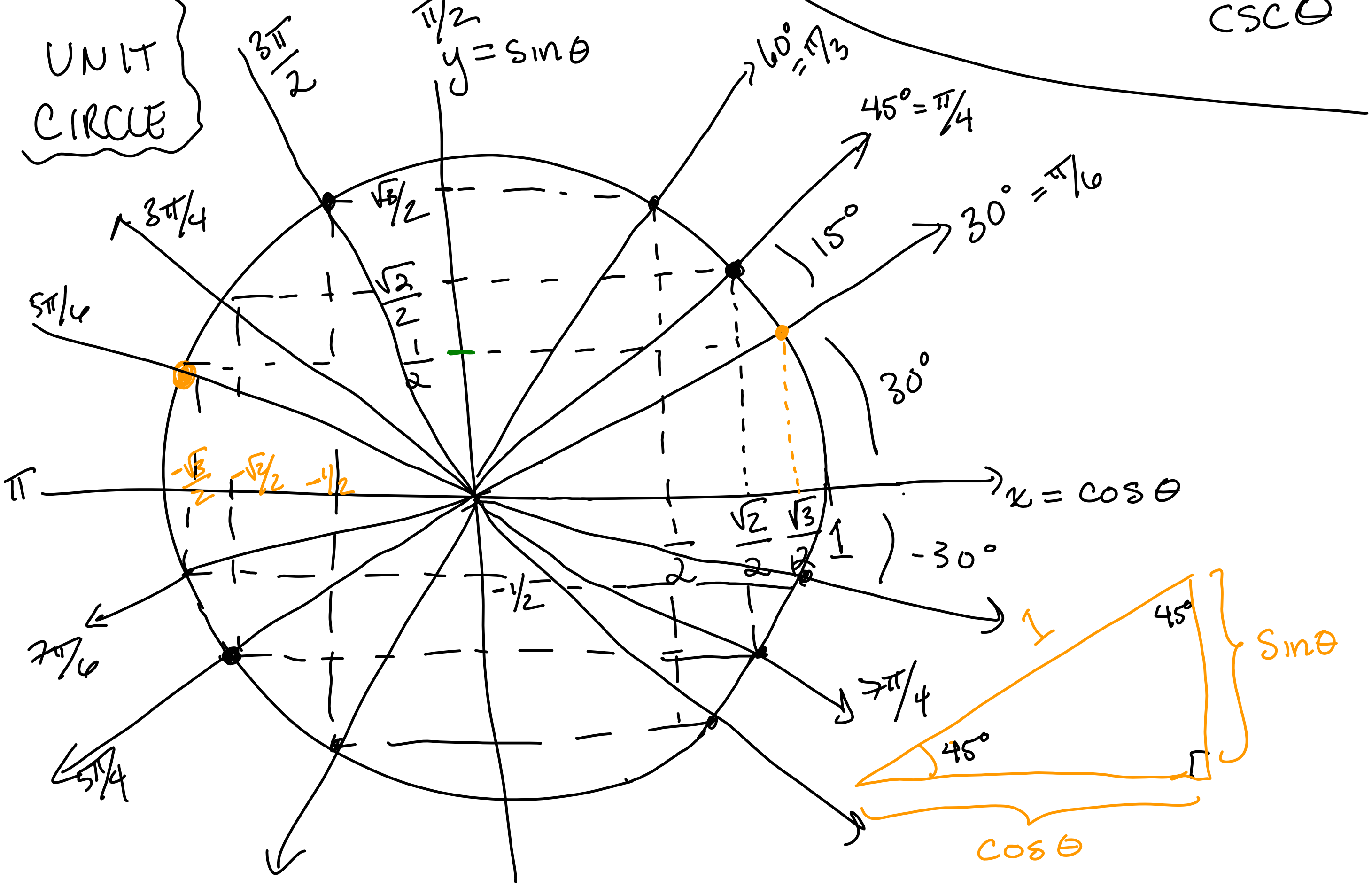
$\sin^{-1}\theta \neq (\sin\theta)^{-1}$

arcsin θ

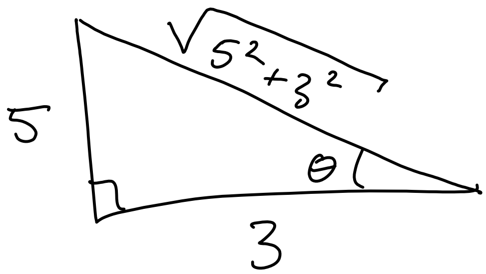
INVERSE sine

$\frac{1}{\sin\theta} = \csc\theta$

UNIT CIRCLE

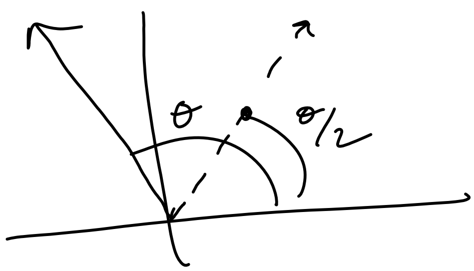


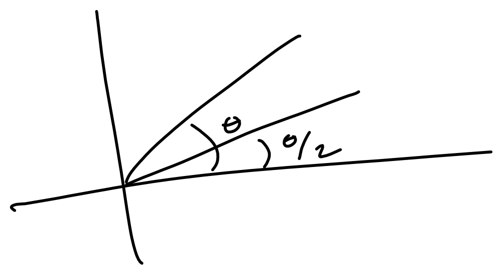
IF $\tan\theta = \frac{5}{3}$, WHAT IS $\sec\theta$?

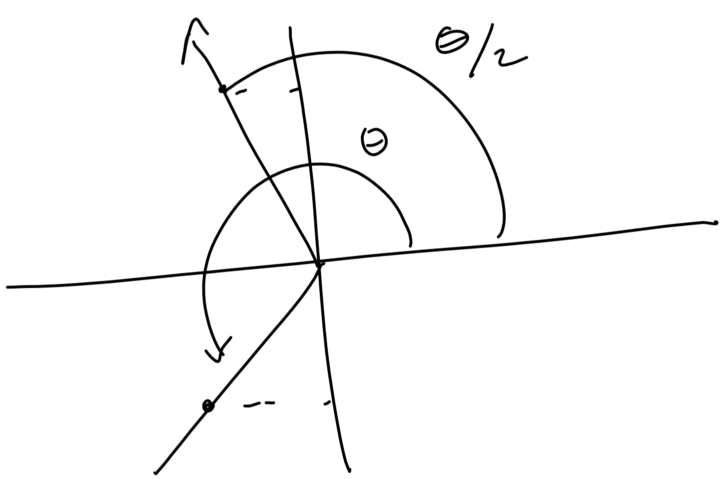


$$\sec\theta = \frac{\text{HYP}}{\text{ADJ}}$$

$$= \frac{\sqrt{34}}{3}$$







$\sin(x+y) = \sin x \cos y + \cos x \sin y$

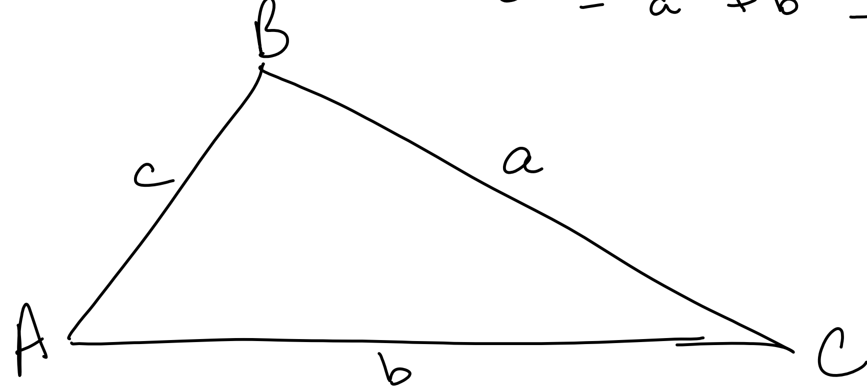
$\sin(x-y) = \sin x \cos y - \cos x \sin y.$

GENERAL $\triangle$ (NON-RIGHT)

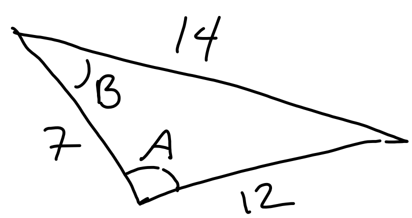
LAW of SINES: $\dfrac{\sin A}{a} = \dfrac{\sin B}{b} = \dfrac{\sin C}{c}$

LAW of COSINES: $a^2 = b^2 + c^2 - 2bc\cos A$

$c^2 = a^2 + b^2 - 2ab\cos C$



7-12-14



$14^2 = 7^2 + 12^2 - 2 \cdot 7 \cdot 12 \cos A$

$3 = -168 \cos A$

$$-\frac{3}{168} = \cos A$$

$$A = 1.588 \text{ radians} \approx 91^\circ$$

$$\frac{\sin A}{14} = \frac{\sin B}{12}$$

$$\frac{0.99}{14} = \frac{\sin B}{12}$$

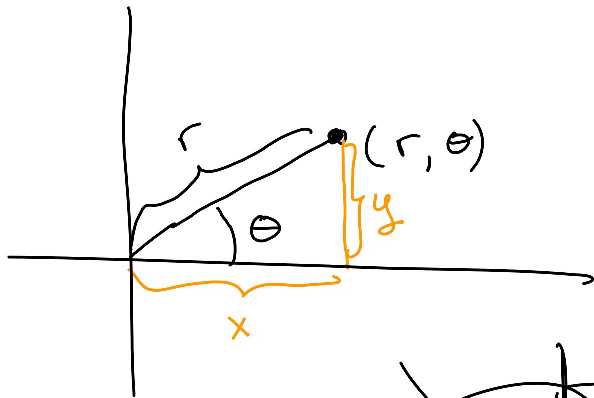
$$\sin B = 0.857$$

$$B = 58.99^\circ$$

$$C = 180 - 58.99 - 91^\circ$$

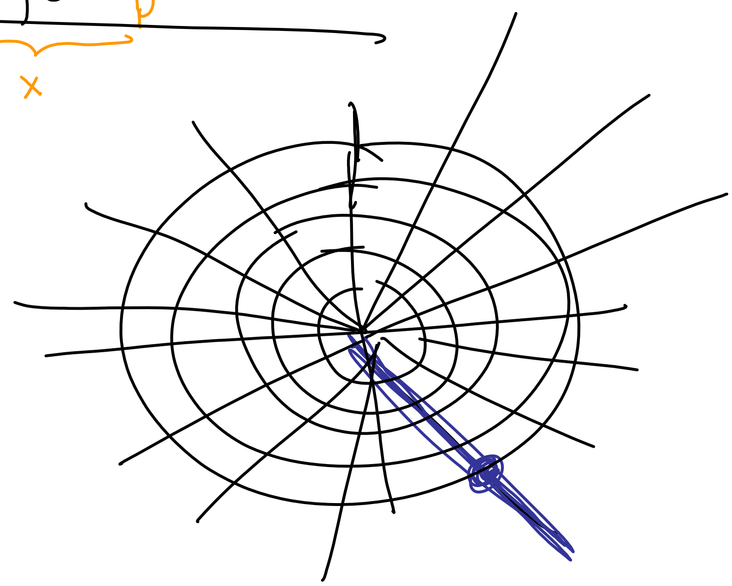
$$= 30.02$$

POLAR COORDINATES



$\cos\theta = \frac{x}{r}$

$\sin\theta = \frac{y}{r}$



$$\left.\begin{aligned} x &= r\cos\theta \\ y &= r\sin\theta \end{aligned}\right\}$$

$r = \sqrt{x^2 + y^2}$

$\tan\theta = \frac{y}{x}$

$x^2 + y^2 = 9$

$r = 9$

Suppose $P = (3, 5\pi/3)$
(IN POLAR COORDS)

FIND CARTESIAN COORDINATES:

$x = 3 \cdot \cos 5\pi/3 = 3 \cdot \frac{1}{2} = \frac{3}{2}$

$y = 3 \sin 5\pi/3 = 3 \cdot \left(\frac{-\sqrt{3}}{2}\right) = \frac{-3\sqrt{3}}{2}$

MORE PROOFS

THM IF P THEN Q.

DIRECT PROOF $P \to S_1 \to S_2 \to \dots \to Q$

PROOF BY CONTRADICTION ASSUME P IS TRUE & Q IS FALSE, WORK FOR A WHILE & ARRIVE AT A CONTRADICTION.

PROOF BY CONTRAPOSITIVE

IMPLICATION $P \to Q$

CONVERSE $Q \to P$

CONTRAPOSITIVE NOT $Q \to$ NOT P. $\Leftarrow$ LOGICALLY EQUIVALENT TO THE IMPLICATION

INVERSE NOT $P \to$ NOT Q $\Leftarrow$ LOGICALLY EQUIV. TO THE CONVERSE.

IMPLICATION

$P \longrightarrow Q$

JOE AT McD'S $\to$ JOE DIDN'T ROB BANK

CONVERSE

JOE DIDN'T ROB BANK $\longrightarrow$ JOE AT McD'S

CONTRAPOSITIVE

not $Q \rightarrow$ not P

JOE ROBBED BANK $\rightarrow$ JOE NOT AT McD's

INVERSE

NOT $P \rightarrow$ NOT Q

JOE NOT AT McD's $\rightarrow$ JOE ROBBED THE BANK.

DIRECT: $P \rightarrow S_1 \rightarrow S_2 \rightarrow \ldots \rightarrow Q$

CONTRAPOSITIVE: NOT $Q \rightarrow S_1 \rightarrow S_2 \cdots \rightarrow$ NOT P

EXAMPLE

THM IF THE SUM OF TWO INTEGERS x, y IS EVEN, THEN ~~THEY~~ HAVE THE SAME PARITY.

P

Q

PROOF ASSUME $\underbrace{\text{NOT } Q}_{x,y \text{ HAVE DIFF. PARITY}}$ & SHOW $\underbrace{\text{NOT } P}_{\text{SUM IS ODD.}}$

WLOG, ASSUME x IS EVEN AND y IS ODD.

THEN WE CAN WRITE $x = 2n$, $y = 2m+1$ FOR SOME INTEGERS n AND m.

THEIR SUM IS $x + y = 2n + 2m + 1$

$= \underbrace{2(n+m)}_{\text{EVEN}} + 1$, AND MUST

BE ODD SINCE $2(n+m)$ IS EVEN.

SINCE NUMBERS OF DIFFERING PARITY SUM TO AN ODD NUMBER, IF THE SUM OF TWO NUMBERS IS EVEN, THEY MUST HAVE THE SAME PARITY. $\square$