PROPERTIES	ALG	GEOM
① EVEN	$f(-x) = f(x)$	SYMMETRIC ACROSS y-AXIS
② ODD	$f(-x) = -f(x)$	POINT SYMMETRY ABOUT THE ORIGIN.

---

ex $f(x) = x^2$ $\quad f(-x) = (-x)^2 = x^2$, so since

$f(x) = f(-x)$, f is EVEN.

ex $f(x) = x^3 + x$ $\quad f(-x) = (-x)^3 + (-x) = -x^3 - x$

$$= -\underbrace{(x^3 + x)}_{f(x)}$$

$$= -f(x)$$

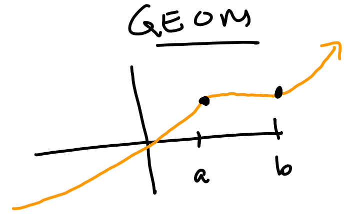
This is ODD.

ex $f(x) = 3x + 7$ $\quad f(-x) = -3x + 7 = -(3x - 7)$

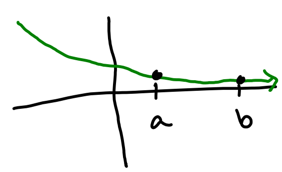
NEITHER EVEN NOR ODD.

MONOTONE	ALG	GEOM

↳ INCREASING: $a \le b \Rightarrow f(a) \le f(b)$



↳ DECREASING: $a \le b \Rightarrow f(a) \ge f(b)$



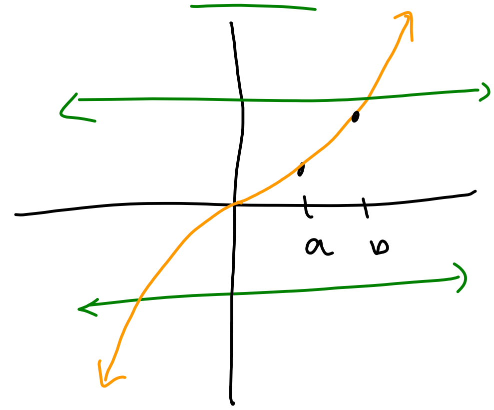
INJECTIVE (1-1)

ALG

IF $a \neq b$ THEN

$f(a) \neq f(b)$.

GEOM



A FUNCTION IS 1-1 IF NO HORIZ. LINE CROSSES ITS GRAPH MORE THAN ONCE.

1-1 FUNCTIONS HAVE INVERSES.

ALGEBRAIC

f AND g ARE INVERSES OF EACHOTHER IF

$f(g(x)) = x$ AND

$g(f(x)) = x$. WE USUALLY WRITE f^{-1}

FOR THE INVERSE OF f.

FINDING THE INVERSE OF A FUNCTION.

ALGEBRAICALLY:
1. SWAP x & y.
2. SOLVE FOR y.

ex $y = 3x - 2 = f(x)$

$x = 3y - 2$

$y = \dfrac{x+2}{3}$ ← I CLAIM THIS IS $f^{-1}(x)$.

CHECK: $f^{-1}(f(x)) = f^{-1}(3x-2) = \dfrac{(\cancel{3}x-\cancel{2})+\cancel{2}}{\cancel{3}} = x.$

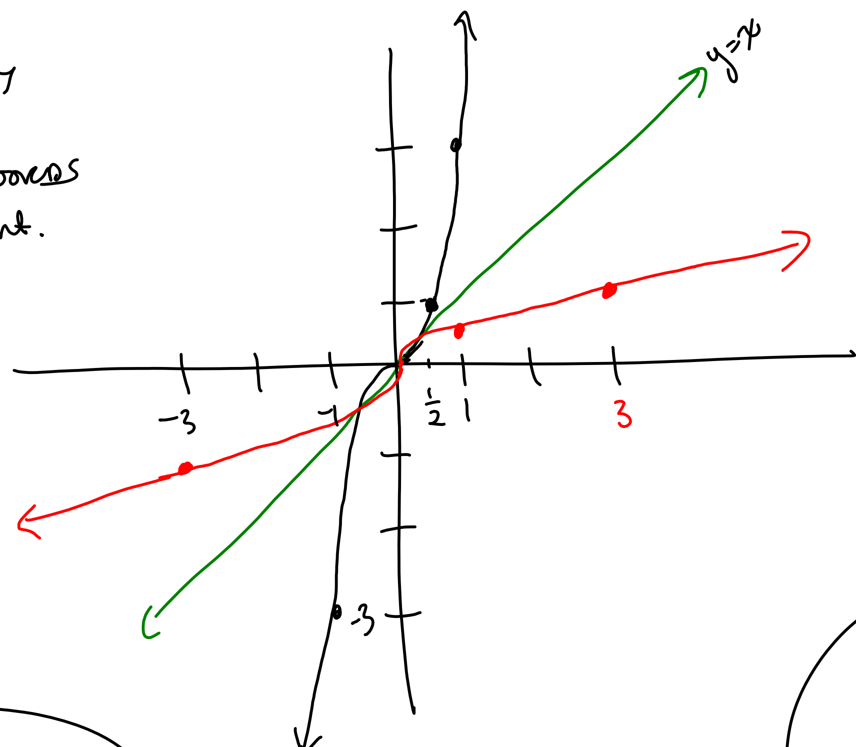
$f(f^{-1}(x)) = f\left(\dfrac{x+2}{3}\right) = \cancel{3}\left(\dfrac{x+\cancel{2}}{\cancel{3}}\right) - \cancel{2} = x$

<u>GEOMETRICALLY</u>

① SWAP x & y COORDS of each point.

<u>OR</u>

② REFLECT ACROSS $y=x$.



TYPES OF FUNCTIONS.

POWER FUNCTION

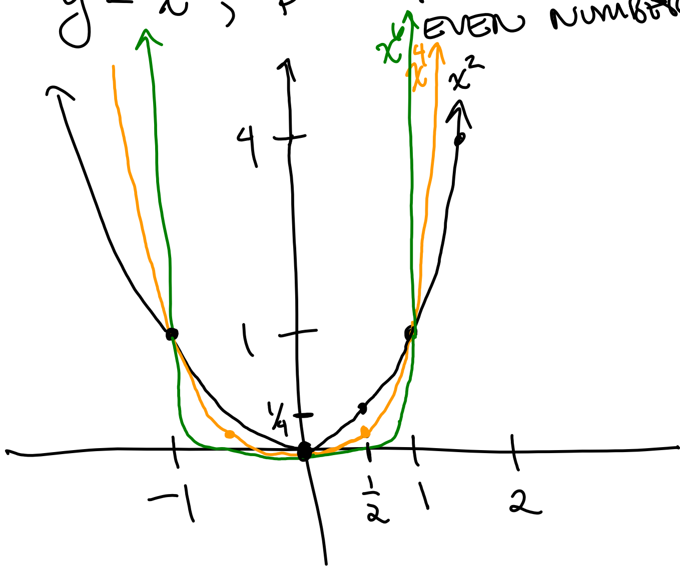
$y = x^p$, p IS A REAL NUMBER $(\neq 0)$

ex $y = x^2$

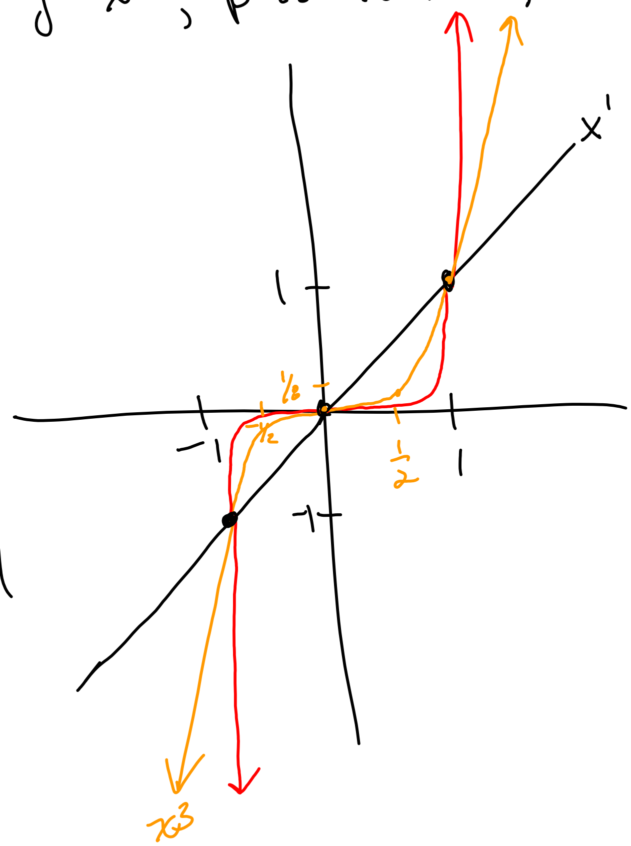
$y = x^{1/3}$

$y = x^{-1/2}$

$y = x^p$, p IS POSITIVE EVEN NUMBER



$y = x^p$, p IS POSITIVE, ODD.



POLYNOMIALS

$y =$ POLYNOMIAL

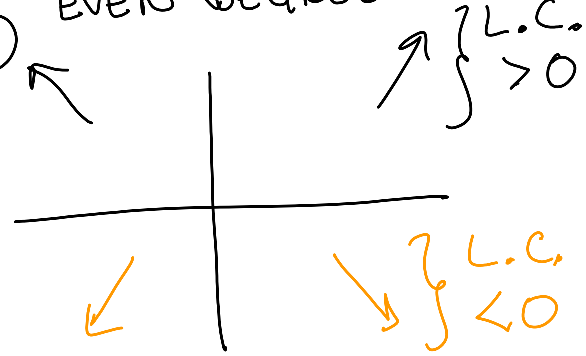
ex $y = 3x^7 - 2x^2 + x - x^5 + 5$

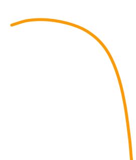
DEGREE

LEADING TERM

LEADING COEFFICIENT.

① EVEN DEGREE

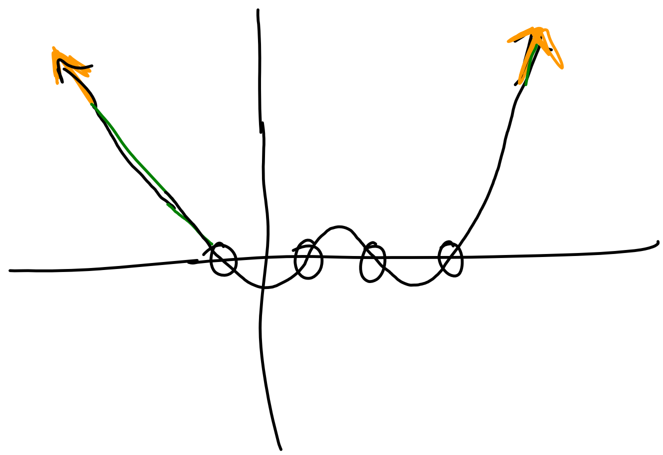




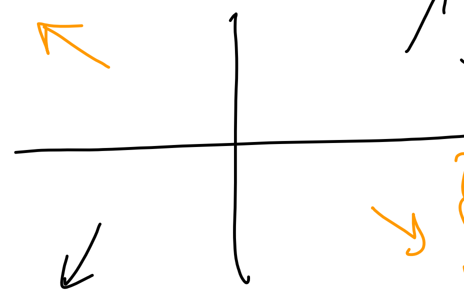
WIGGLES

OF CHANGES IN DIRECTION IS NO MORE THAN DEG − 1.

ex $y = 3x^4 - 2x^3 + \frac{x^2}{5} - 6x + 9$

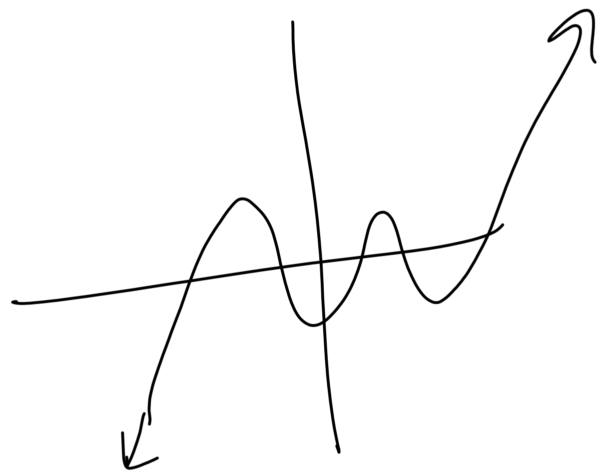


ODD DEGREE



L.C. > 0

L.C. < 0



RATIONAL FUNCTION

$$f(x) = \frac{P(x)}{Q(x)}$$

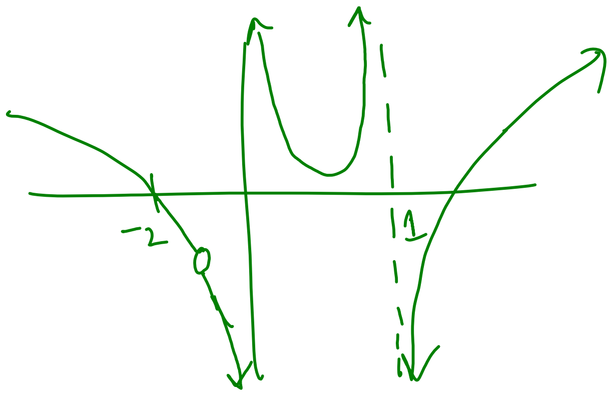
POLYNOMIALS.

$$f(x) = \frac{(x^2+1)(3x+6)(x+1)}{x\,(x-1)(x+1)}$$

HOLE IN GRAPH AT $x = -1$.

ROOTS OF NUMERATOR ARE x-INTERCEPTS

ROOTS OF DENOMINATOR ARE VERTICAL ASYMPTOTES.

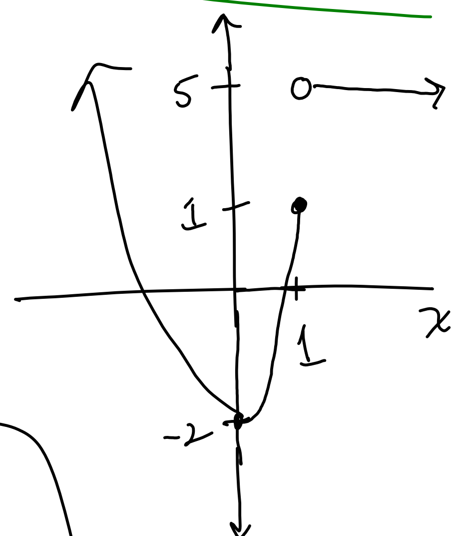


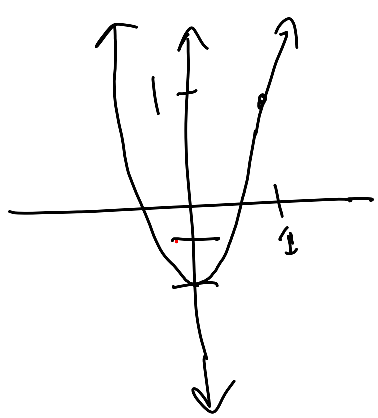
← NOT CLAIMED AS THE GRAPH of $f(x)$ ABOVE

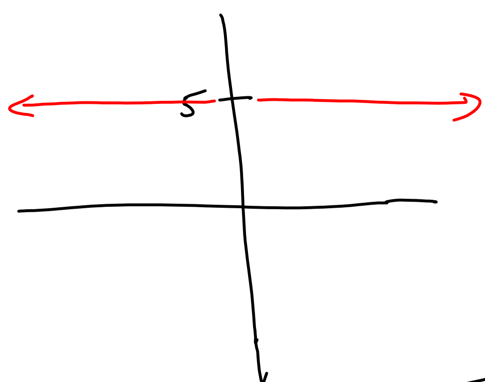
PIECEWISE FUNCTION

ex

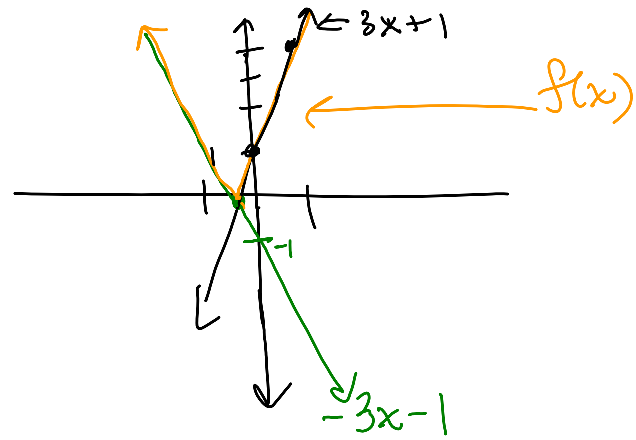
$$f(x)=\begin{cases}3x^2-2 & \text{IF } x\le 1\\ 5 & \text{IF } x>1\end{cases}$$







$$f(x)=|3x+1|=\begin{cases}3x+1 & \text{IF } 3x+1\ge 0 \quad x\ge -1/3\\ -(3x+1) & \text{IF } 3x+1<0 \quad x<-1/3\end{cases}$$



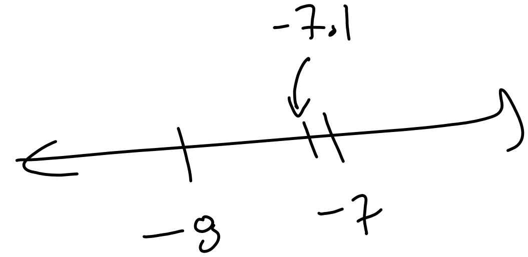
BIGGEST SMALLER INTEGER

$f(x) = [x]$ "GREATEST INTEGER FUNCTION"

$f(3.2) = [3.2] = 3$

$f(3.5) = 3$

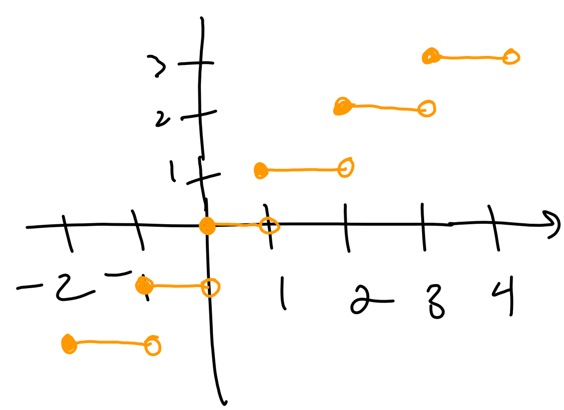
$f(-7.1) = [-7.1] = -8$



$\vdots$

$$f(x) = \begin{cases} 3 & \text{IF } 3 \le x < 4 \\ 4 & \text{IF } 4 \le x < 5 \\ 5 & \text{IF } 5 \le x < 6 \end{cases}$$

$\vdots$



PROOFS! (LAST TYPE)

MATHEMATICAL INDUCTION.

USED TO PROVE SOME TRUE ABOUT ALL

NUMBERS $\geq a$.

STARTING POINT.

TWO PARTS:

1. BASE CASE - PROVE IT'S TRUE FOR a.
2. INDUCTIVE STEP - IF (IT'S TRUE ABOUT n) THEN (IT'S TRUE ABOUT $n+1$).

EXAMPLE

THM FOR ANY POSTIVE INTEGER n,

$$1+2+\ldots+n = \frac{n(n+1)}{2}.$$

PROOF. WE PROCEDE BY INDUCTION.

BASE CASE: CHECK THAT THE THEOREM HOLDS FOR $n=1$:

$$1 = \frac{1(1+1)}{2} = 1.$$

INDUCTIVE STEP. ASSUME IT'S TRUE ABOUT n.

$$1+\dots+n=\frac{n(n+1)}{2}.$$

INDUCTIVE HYPOTHESIS

WE WANT TO USE

WANT TO SHOW THAT

$$1+2+\dots+n+(n+1)=\frac{(n+1)((n+1)+1)}{2}.$$

TO SHOW

$$\underbrace{1+2+\dots+n}_{=\frac{n(n+1)}{2}\text{ BY I.H.}}+(n+1)=\frac{n(n+1)}{2}+(n+1)=\frac{n(n+1)+2(n+1)}{2}$$

$$=\frac{n^2+n+2n+2}{2}=\frac{n^2+3n+2}{2}$$

$$=\frac{(n+1)(n+2)}{2}=\frac{(n+1)((n+1)+1)}{2} \quad \blacksquare$$