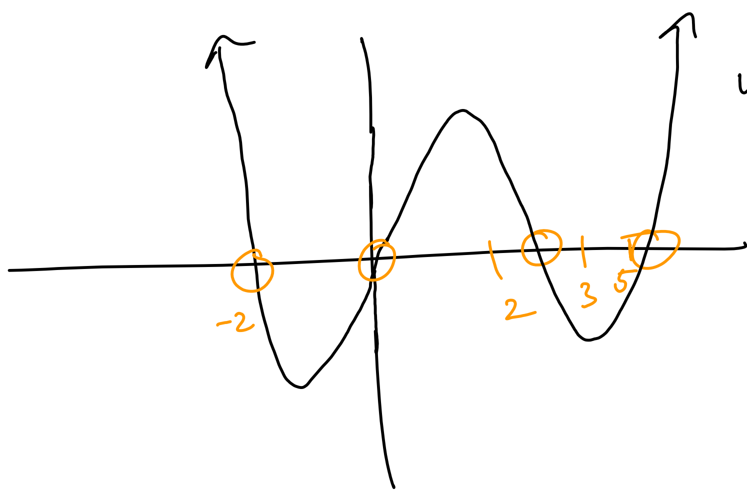
EXERCISES FROM YESTERDAY (ON FUNCTIONS)

S.R. #2

$$f(x) = \frac{x^4}{2} - 3x^3 - x^2 + 15x + \underline{\underline{c}}$$

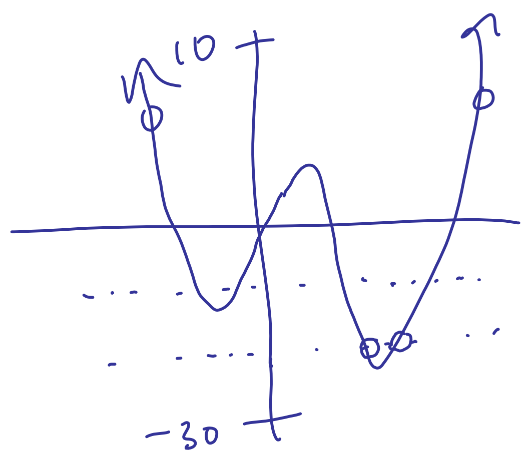
a)



WITH $c=0$

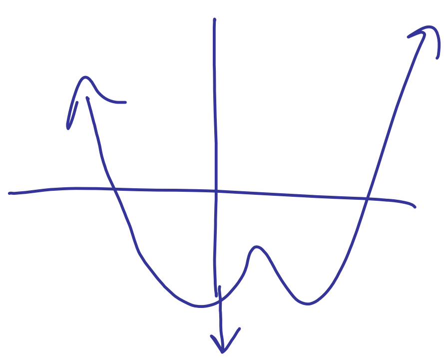
THE GRAPH IS AS AT LEFT, AND SINCE IT HAS 4 DISTINCT x-INTERCEPTS, AND SO 4 REAL ROOTS.

b)



WITH $c=30$, THE GRAPH WILL SHIFT UP HIGH ENOUGH SO THAT IT DOESN'T TOUCH THE AXIS AND THUS HAS NO ROOTS.

c)



IF $c=-20$, THE GRAPH DOWN ... etc.

#2 STRUCT. RESPONSE.

100 TREES, AVG YIELD PER TREE IS 70 ORANGES.

EACH ADD'L TREE CAUSES THE AVG YIELD PER TREE TO DECREASE BY 2.

(a) TOTAL YIELD = # TREES × # ORANGES PER TREE = 100 × 70 = 7000 ORANGES

(b) IF ADD 3 TREES, AVG YIELD PER TREE IS 64.

TOTAL YIELD = 103 × 64 = 6592,

SO TOTAL YIELD DECREASES BY 408.

(c) EXPRESS $\underbrace{\quad\quad}_{y}$ AS AS A FUNCTION OF $\underbrace{\quad\quad}_{x}$.

FIND $y = f(x)$.

LET y REPRESENT THE TOTAL YIELD.

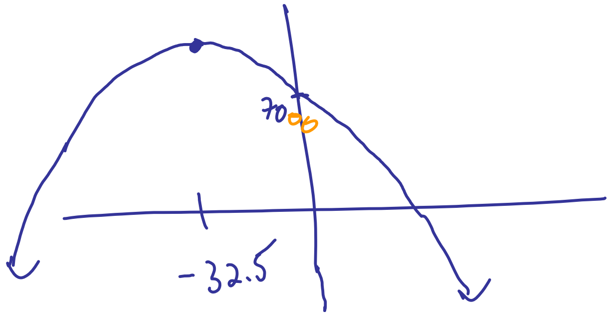
LET x BE THE NUMBER OF ADD'L TREES PLANTED.

$$y = \left(\#\text{ of TREES}\right)\left(\begin{matrix}\text{YIELD}\\\text{PER}\\\text{TREE}\end{matrix}\right) = (100+x)(70-2x)$$

$$y = 7000 - 130x - 2x^2$$

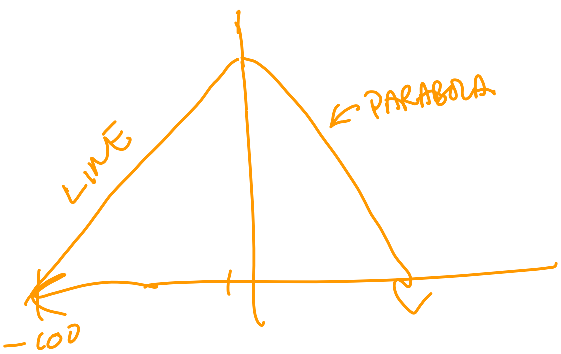
(d) THE VERTEX OF THIS PARABOLA CORRESPONDS TO

THE MAXIMUM SINCE IT OPENS DOWN.



ASSUMING AVG INCREASES IF REMOVES TREES, HE SHOULD KEEP ONLY 67 TREES TO MAXIMIZE HIS YIELD.

IF WE ASSUME HE WILL NOT REMOVE TREES HE SHOULD NOT PLANT ANY ADD'L ONES.



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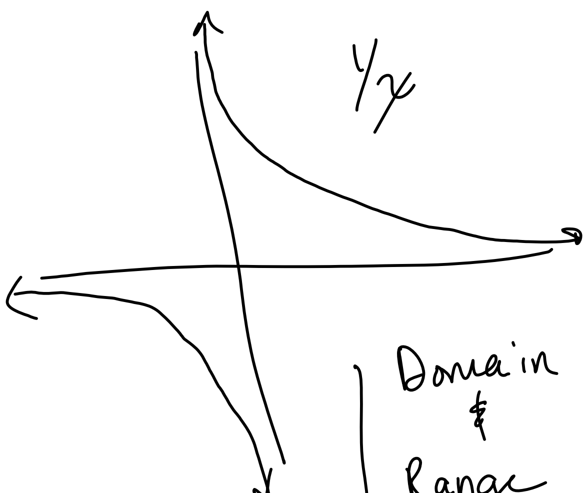
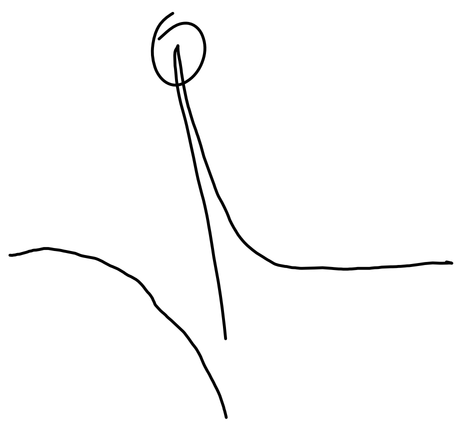




Domain & Range are both ALL ℝ EXCEPT 0.

$(-\infty, 0) \cup (0, \infty)$

$\mathbb{R} - \{0\}$

$f(x) = \frac{1}{\sqrt{3x-2}}$

FIND DOMAIN:

$3x-2 \neq 0$, $3x-2 \nless 0$, so

$3x-2>0$

$x > \frac{2}{3}$; $(\frac{2}{3}, \infty)$

RANGE is $\{y>0\}$

$(0,\infty)$

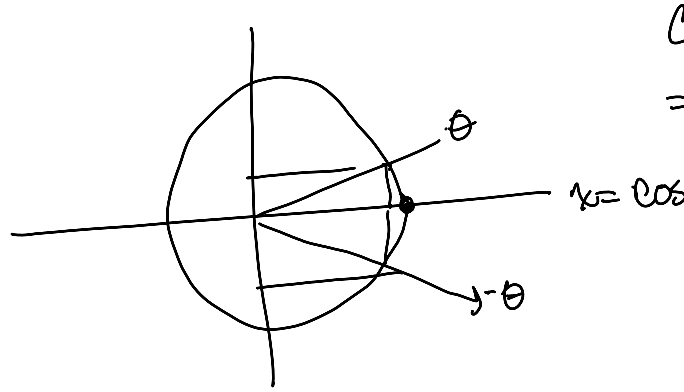
$3x-2 = \frac{1}{4}$

$3x = \frac{9}{4}$

$x = \frac{3}{4}$

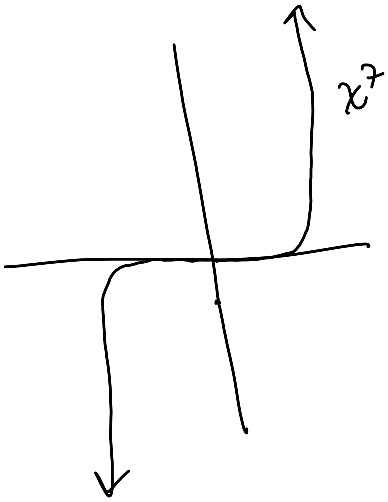
$\sqrt{r^3} = (r^3)^{1/2} = r^{3/2} = \sqrt{r^2 \cdot r} = r\sqrt{r}$

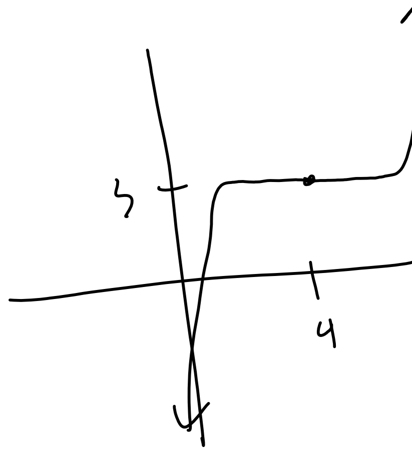
$= (\sqrt{r})^3 =$



$\cos(-\theta)$

$= \cos(\theta)$





$(f+g)(x) = \underline{f(x) + g(x)}$ $\qquad$ $(f+g)(4) = f(4) + g(4)$

$9 + 6 = 15$

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10. $g(x) = 3x^2 - 7$

$f(x) = 5\sin x$

- TRANSCENDENTAL FUNCTIONS

(ie., NOT ALGEBRAIC — MEANS BUILT FROM POLYNOMIALS USING $+, \cdot, -, \div, \sqrt[n]{\ }$, exponents – but not exponentials)

(1) EXPONENTIAL FUNCTIONS

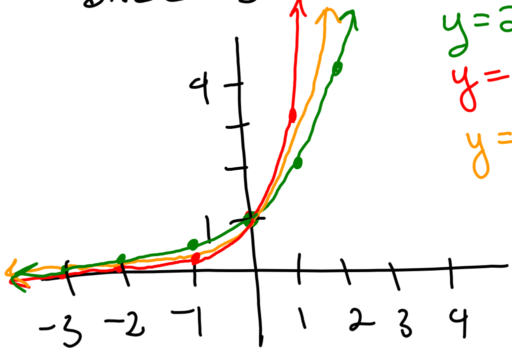
$y = b^x$ (COMPARE TO POWER FUNCTIONS $y = x^p$) A NUMBER

A CONSTANT, REAL, POSITIVE NUMBER.

ex $y = 2^x$; $y = \left(\frac{1}{4}\right)^x = 4^{-x}$

$y = e^x$

BASE $b > 1$



$y = 2^x$

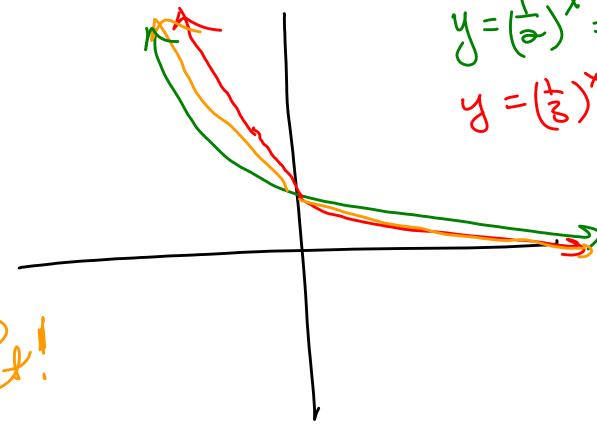
$y = 3^x$

$y = e^x$

EXPONENTIAL GROWTH FUNCTIONS

reflect!

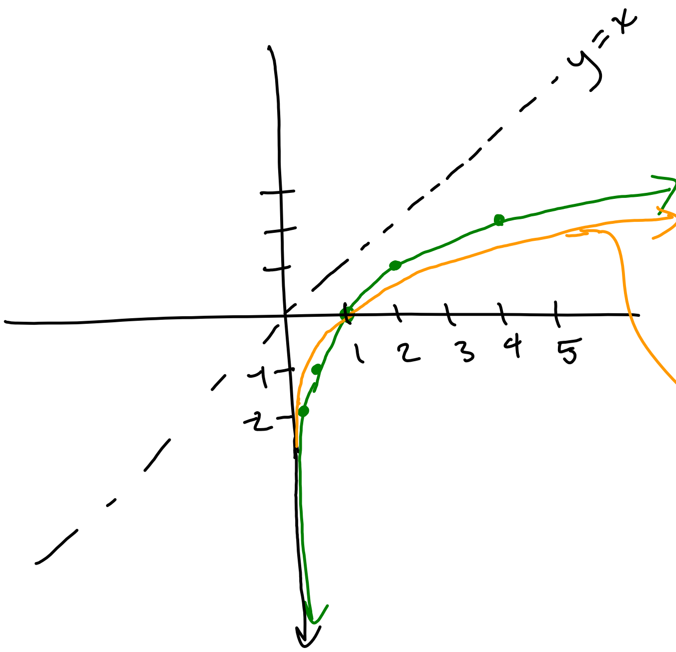
BASE $b < 1$



$y = \left(\frac{1}{2}\right)^x = 2^{-x}$

$y = \left(\frac{1}{3}\right)^x = 3^{-x}$

EXPONENTIAL DECAY



$y = \log_2 x$ <u>means</u> $x = 2^y$ $\leftarrow$ INVERSE OF $y = 2^x$

$y = \log_e x = \ln x$ <u>means</u> $x = e^y$

LOGARITHMS

$y = \log_b x$ <u>MEANS</u> $x = b^y$.

b ← POSITIVE REAL NUMBER

$y = \ln x$ means $x = e^y$.

LOG RULES (TRUE FOR ANY BASE)

$$\log_b x + \log_b y = \log_b(x \cdot y)$$

$$\log_b x - \log_b y = \log_b\left(\frac{x}{y}\right)$$

$$\log_b(x^y) = y \log_b(x)$$

$\log_x k = m$ *means* $x^m = k$

SOLVE $\quad 8^x = 5^{2x^2-1}$

$$\ln(8^x) = \ln\left(5^{2x^2-1}\right)$$

$$x \ln 8 = (2x^2-1)\ln 5$$

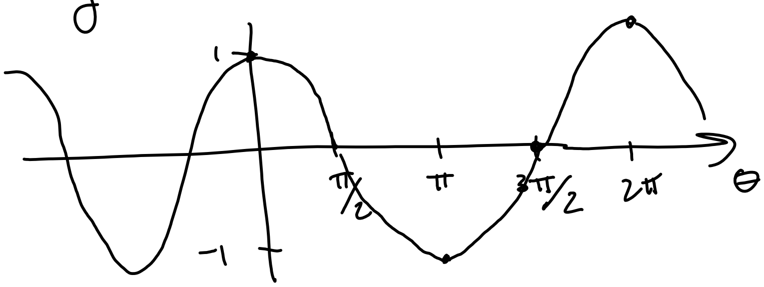
$$\Rightarrow x \ln 8 = (2\ln 5)x^2 - \ln 5$$

$$0 = \underbrace{(2\ln 5)}_{a}x^2 \underbrace{-(\ln 8)}_{b}x \underbrace{-\ln 5}_{c}$$

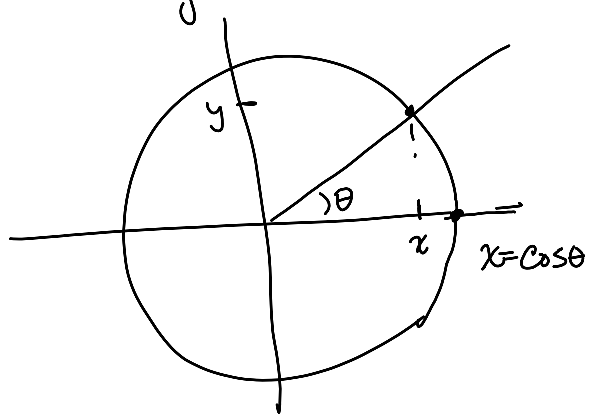
USE QUAD. FORMULA.

TRIGONOMETRIC FUNCTIONS

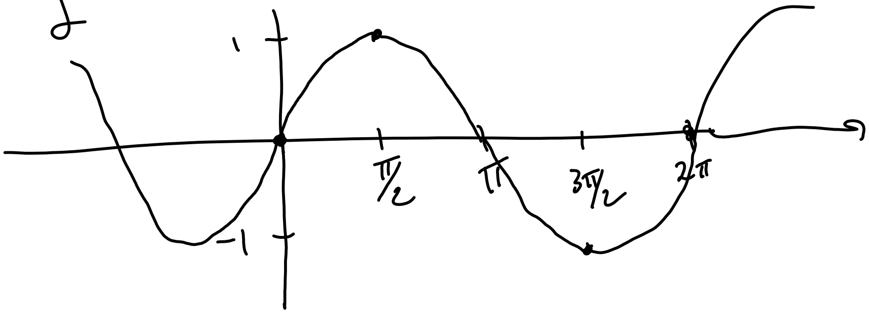
$y = \cos\theta$



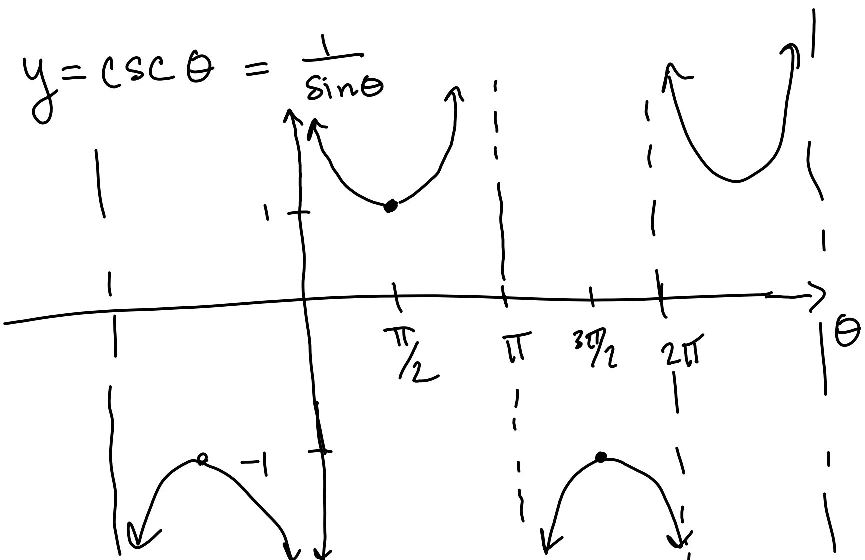
$y = \sin\theta$



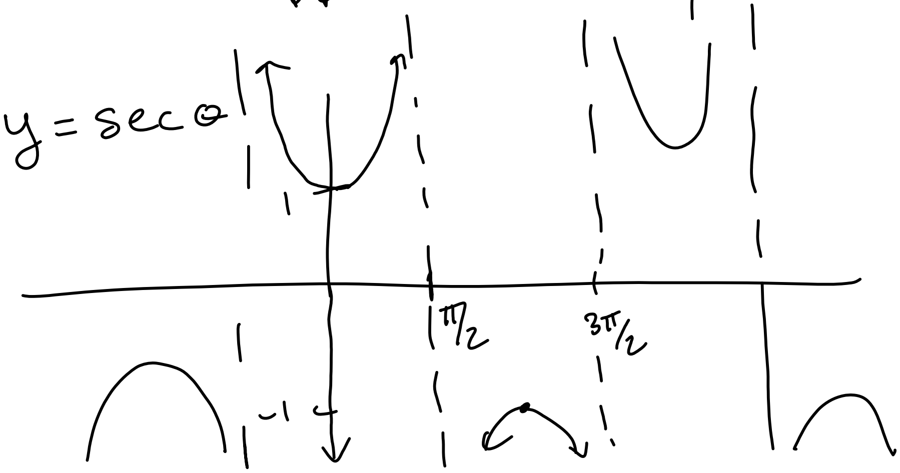
$y = \sin\theta$



$y = \csc\theta = \frac{1}{\sin\theta}$



$y = \sec\theta$



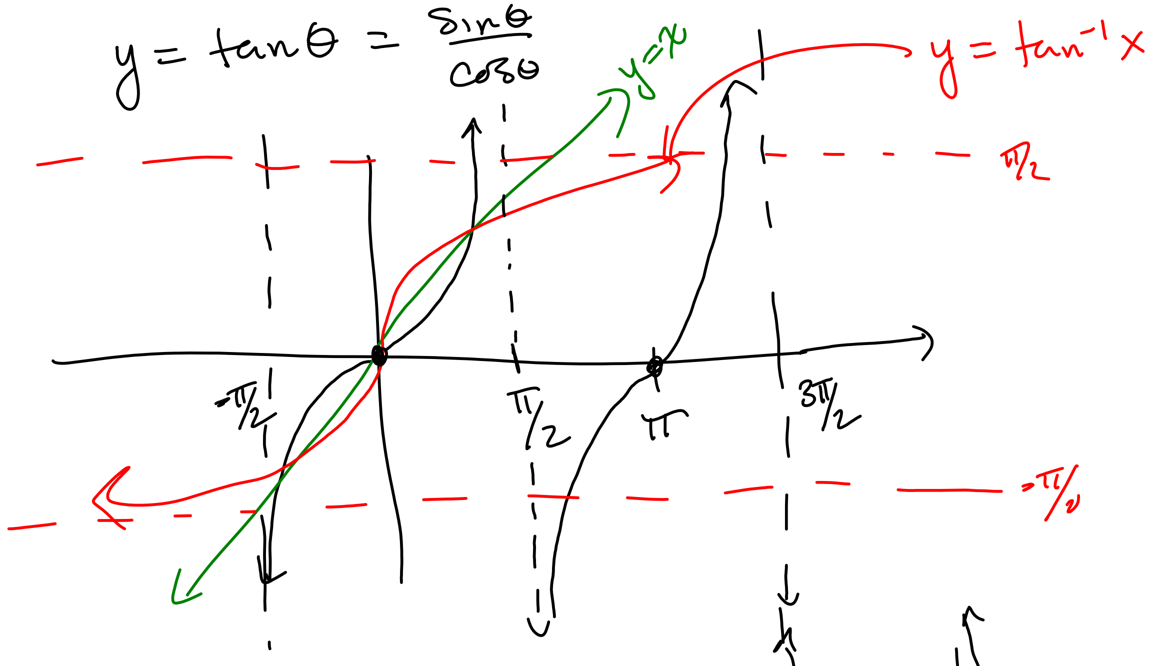
$$\frac{1}{\sin\left(\theta+\frac{\pi}{2}\right)} = \frac{1}{\cos\theta}$$

$$\csc\left(\theta+\frac{\pi}{2}\right) = \sec\theta$$

$$\cos\left(\theta-\frac{\pi}{2}\right) = \sin\theta$$

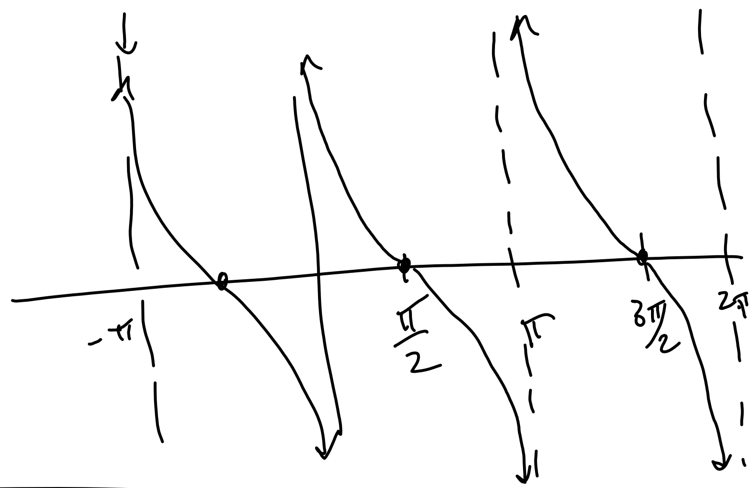
$$\cos\left(\theta+\frac{\pi}{2}\right) = -\sin\theta$$

$y = \tan\theta = \dfrac{\sin\theta}{\cos\theta}$



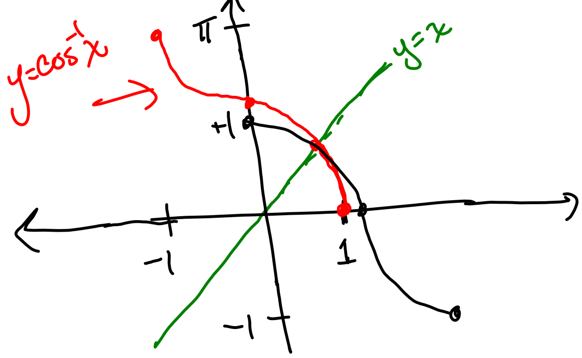
$y = \cot\theta = \dfrac{1}{\tan\theta}$

$= \dfrac{\cos\theta}{\sin\theta}$



FOR $y = \cos^{-1}x$, DOMAIN $[-1,1]$

WE RESTRICT DOMAIN OF $\cos\theta$ TO $[0,\pi]$



$f(x)$ $\quad$ $f^{-1}(x)$