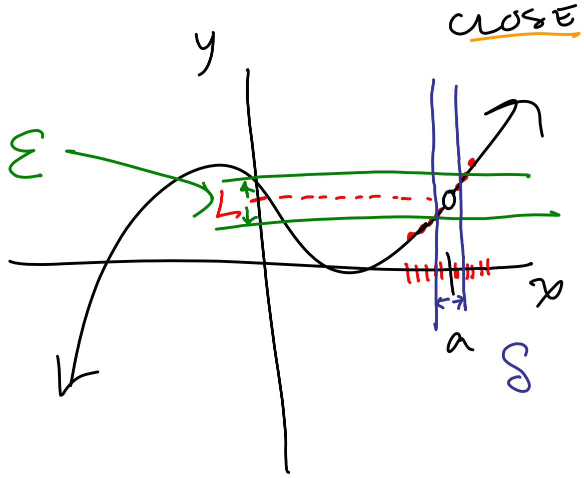
LIMIT

INTUITION: SAY $\lim_{x\to a} f(x) = L$ IF AS THE x-VALUES GET REALLY CLOSE TO a, THE $y(=f(x))$-VALUES GET REALLY CLOSE TO L.



Defn $\lim_{x\to a} f(x) = L$ IF FOR EACH $\varepsilon > 0$, THERE IS A $\delta > 0$ SO THAT WHENEVER $0 < |x-a| < \delta$, THEN

$$|f(x) - L| < \varepsilon.$$

Limit Laws

— ASSUME $\lim_{x\to a} f(x)$ & $\lim_{x\to a} g(x)$ EXIST.

① $\lim_{x\to a} k = k$

② $\lim_{x\to a} x = a$

③ $\lim_{x\to a} k f(x) = k \lim_{x\to a} f(x)$ | ex $\lim_{x\to 3} 5x = 5 \lim_{x\to 3} x = 5\cdot 3 = 15$

④ $\lim_{x\to a} f(x) \pm g(x) = \lim_{x\to a} f(x) \pm \lim_{x\to a} g(x)$ | ex $\lim_{x\to 2} x + 7 = \lim_{x\to 2} x + \lim_{x\to 2} 7 = 2 + 7 = 9$

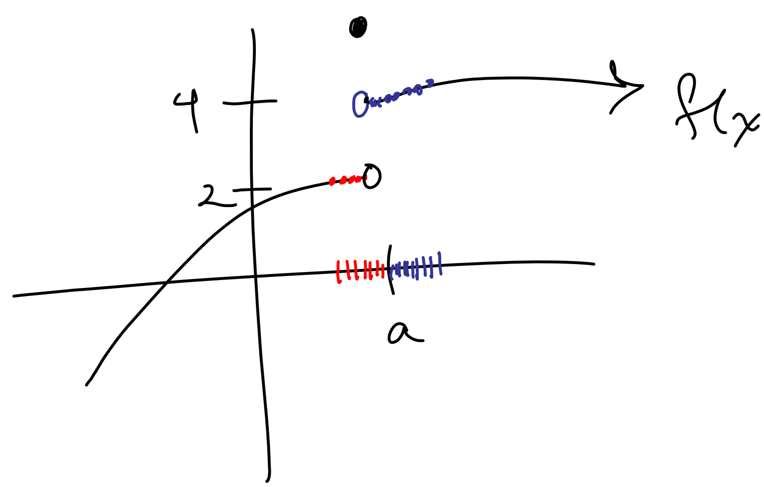
⑤ $\lim_{x \to a} f(x) \cdot g(x) = \lim_{x \to a} f(x) \cdot \lim_{x \to a} g(x)$ $\Bigg\{$ ex $\lim_{x \to 2} x^2 = \left(\lim_{x \to 2} x\right) \cdot \left(\lim_{x \to 2} x\right) = 2 \cdot 2 = 4$

⑥ $\lim_{x \to a} \dfrac{f(x)}{g(x)} = \dfrac{\lim_{x \to a} f(x)}{\underbrace{\lim_{x \to a} g(x)}_{\text{THIS CAN'T BE ZERO}}}$ $\Bigg\{$ ex $\lim_{x \to 4} \dfrac{5}{x} = \dfrac{\lim_{x \to 4} 5}{\lim_{x \to 4} x} = \dfrac{5}{4}$

⑦ $\lim_{x \to a} \left(f(x)\right)^p = \left(\lim_{x \to a} f(x)\right)^p$

ex $\lim_{x \to 5} \dfrac{x^2 - 3x + 1}{2\sqrt{x-1}} = \dfrac{\lim_{x \to 5}(x^2 - 3x + 1)}{2\sqrt{\lim_{x \to 5}(x-1)}} = \dfrac{5^2 - 3 \cdot 5 + 1}{2 \cdot \sqrt{4}}$

LEFT & RIGHT LIMITS



$\lim_{x \to a^-} f(x) = 2$ ← LEFT

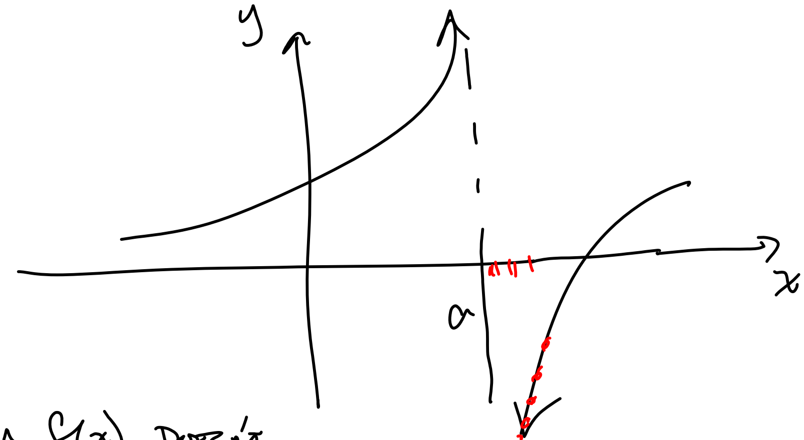
$\lim_{x \to a^+} f(x) = 4$ ← RIGHT

$\lim_{x \to a} f(x)$ DOES NOT EXIST.

INFINITE LIMITS

$$\lim_{x\to a^-} f(x) = \infty$$

$$\lim_{x\to a^+} f(x) = -\infty$$

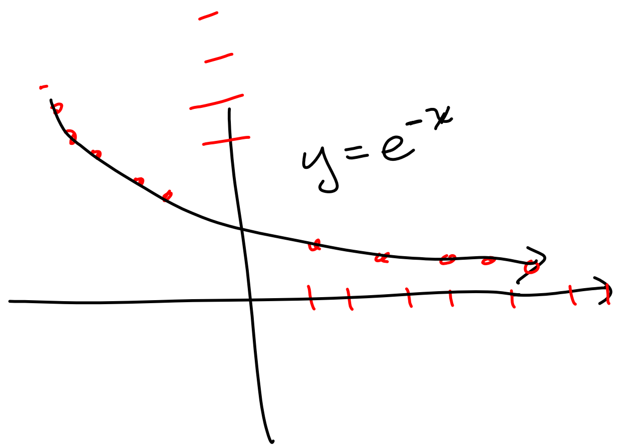


$\lim_{x\to a} f(x)$ DOESN'T EXIST.

LIMITS AT INFINITY

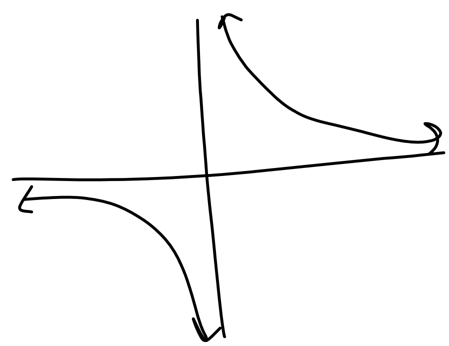
$$\lim_{x\to \infty} f(x) = 0$$

$$\lim_{x\to -\infty} f(x) = \infty$$



$$\lim_{x\to\infty} \frac{1}{x} = 0$$

$$\lim_{x\to\infty} \frac{1}{x^p} = 0 \quad , p>0$$



$$\lim_{x\to\infty} \frac{(x^2-3)}{(2x^4-9x)} \cdot \frac{\frac{1}{x^4}}{\frac{1}{x^4}} = \lim_{x\to\infty} \frac{\frac{1}{x^2} - \frac{3}{x^4}}{2 - \frac{9}{x^3}} = \frac{0-0}{2-0} = \frac{0}{2}$$

$$= 0$$

TRICK FOR FINDING $\lim_{x \to \infty}$ (RATIONAL FCTNS)

MULTIPLY NUM & DEN BY $\frac{1}{x^{(\text{HIGEST POWER of } x \text{ IN DEN})}}$