LIMITS (EXERCISE 1)

a) $\lim\limits_{x \to 1} \dfrac{x^2+x-2}{x^2-1} = \lim\limits_{x \to 1} \dfrac{(x+2)\cancel{(x-1)}}{(x+1)\cancel{(x-1)}} = \dfrac{3}{2}$

d) $\lim\limits_{x \to 2} \dfrac{3x^2-9x+6}{x^2-4} = \lim\limits_{x \to 2} \dfrac{3(x-1)\cancel{(x-2)}}{(x+2)\cancel{(x-2)}} = \dfrac{3}{4}$

e) $\lim\limits_{x \to \infty} \dfrac{3x^2-9x+6}{x^2-4} \cdot \dfrac{\frac{1}{x^2}}{\frac{1}{x^2}} =$

$\lim\limits_{x \to \infty} \dfrac{3x-3}{x+2} \cdot \dfrac{\frac{1}{x}}{\frac{1}{x}} =$

TIP:
IF WHEN YOU EVALUATE A LIMIT OF $\dfrac{*}{+}$ AND YOU GET

$\dfrac{0}{0} \to$ TRY TO CANCEL SOMETHING

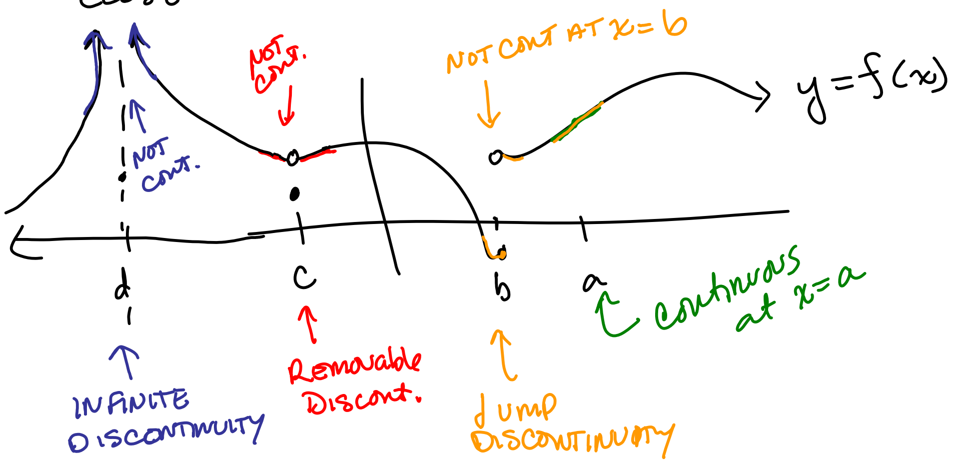
$\dfrac{\neq 0}{0} \to \pm\infty$ LIMIT?

$\dfrac{0}{\neq 0} \to$ THIS IS 0.

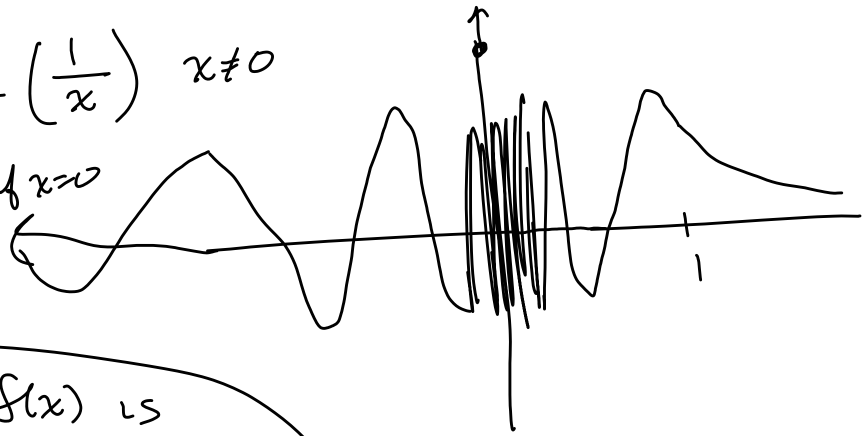
$\dfrac{\infty}{\infty} \to$ TRY TO CANCEL

CONTINUITY!

(INTUITION)
A FUNCTION $y = f(x)$ IS CONTINUOUS AT $x = a$ IF YOU CAN DRAW THE PICTURE OF THE graph CLOSE TO a WITHOUT PICKING UP YOUR PEN.



$$y = \begin{cases} \sin\left(\frac{1}{x}\right) & x \neq 0 \\ 3 & \text{if } x = 0 \end{cases}$$



COSCILLATING DISCONTINUITY.

Def $y = f(x)$ is continuous at $x = a$

① $\lim_{x \to a} f(x) = L$

② $f(a)$ EXISTS

③ $L = f(a)$

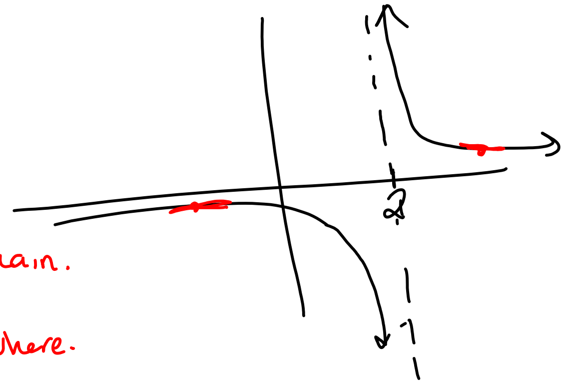
$$\lim_{x \to a} f(x) = f(a)$$

FUNCTIONS LISTED HERE ARE CONTINUOUS ON THEIR DOMAINS

POLYNOMIALS, RATIONAL FUNCTIONS, exponential functions, root functions, log functions, trig functions

cos & sin are cont. everywhere

ex $y = \dfrac{1}{x-2}$



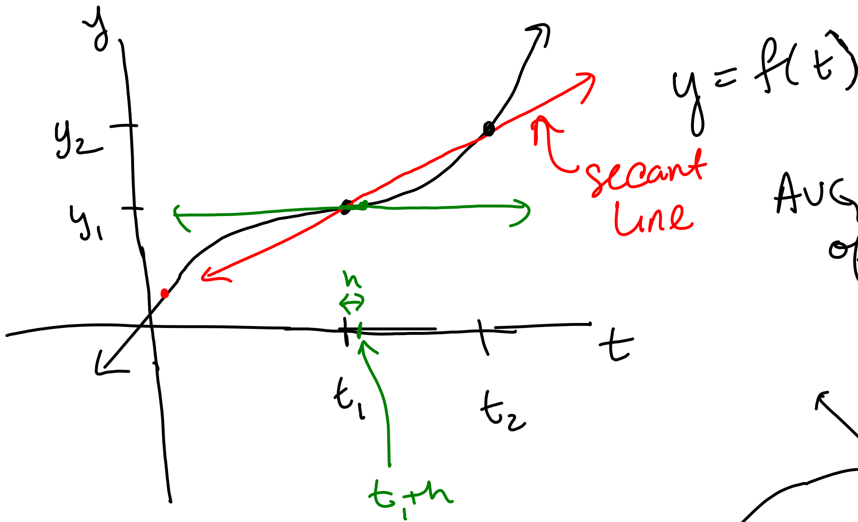
Continuous on its domain.

NOT continuous everywhere.

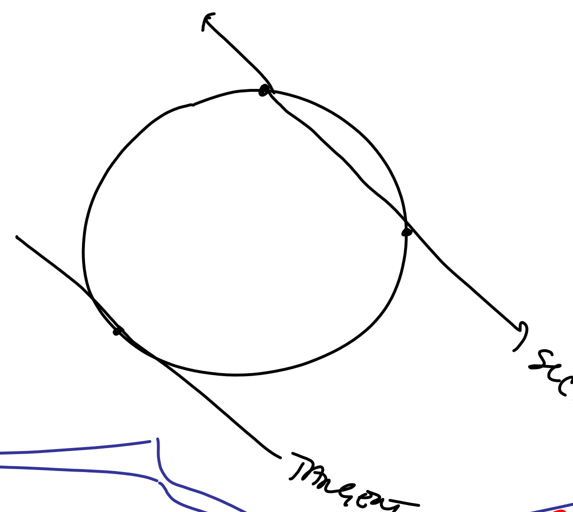
$$\lim_{x \to 1} 3\cos(x-1) + \ln x = 3\cos(0) + \ln 1$$

$$= 3 \cdot 1 + 0 = 3$$

DERIVATIVES



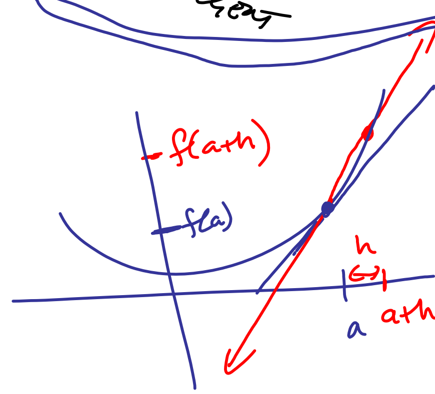
AVG RATE of change $\dfrac{y_2 - y_1}{t_2 - t_1}$



Instantaneous rate of change $\displaystyle\lim_{t_2 \to t_1} \frac{y_2 - y_1}{t_2 - t_1}$

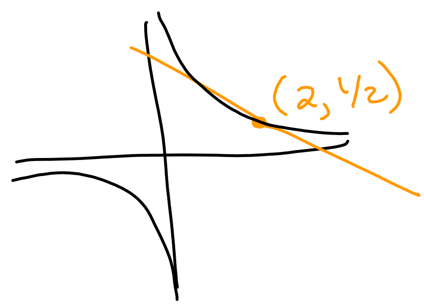
Defn of the line tangent to $y = f(x)$ at $x = a$: THE LINE THAT PASSES THROUGH $(a, f(a))$ AND HAS SLOPE

$$\underline{f'(a) = m} = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}.$$



SLOPE of tangent line at $x=a$ $\equiv$ Derivative of $f(x)$ at $x=a$ $\equiv$ Instantaneous Rate of change at $x=a$.

FIND THE EQN OF THE LINE TANGENT TO $y=f(x)=\frac{1}{x}$ AT $a=2$.



$$f'(2)=\lim_{h\to 0}\frac{f(2+h)-f(2)}{h}$$

$$=\lim_{h\to 0}\frac{\frac{1}{2+h}-\frac{1}{2}}{h}$$

$$=\lim_{h\to 0}\frac{2-(2+h)}{2(2+h)}\cdot\frac{1}{h}$$

$$=\lim_{h\to 0}\frac{-\cancel{h}}{2(2+h)\cancel{h}}=\lim_{h\to 0}\frac{-1}{2(2+h)}=-\frac{1}{4}$$

SLOPE

$$f(2+h)=\frac{1}{2+h}$$

$$f(\square)=\frac{1}{\square}$$

LINE $y-\frac{1}{2}=-\frac{1}{4}(x-2)$

WAYS TO CHEAT (aka the derivative rules)

1. IF $f(x)=k$, THEN $f'(x)=0$
2. IF $f(x)=x$, THEN $f'(x)=1$
3. IF $f(x)=x^p$, THEN $f'(x)=p\,x^{p-1}$

ex $f(x)=\sqrt{x}=x^{1/2}$

$f'(x)=\frac{1}{2}x^{-1/2}=\frac{1}{2\sqrt{x}}$

4. IF $f(x)=k\cdot g(x)$, THEN $f'(x)=k\cdot g'(x)$

ex $f(x)=5\cdot x^2$, THEN $f'(x)=5\cdot 2x$

5. IF $f(x)=g(x)\pm h(x)$, THEN $f'(x)=g'(x)\pm h'(x)$

ex $f(x)=5x^4-3\sqrt{x}$, THEN $f'(x)=20x^3-\frac{3}{2}x^{-1/2}$

6. IF $f(x)=g(x)\cdot h(x)$, THEN $f'(x)=g'(x)h(x)+h'(x)g(x)$

7. IF $f(x)=\dfrac{g(x)}{h(x)}$, THEN $f'(x)=\dfrac{g'(x)h(x)-h'(x)g(x)}{(h(x))^2}$

8. IF $f(x)=g(h(x))$, THEN $f'(x)=g'(h(x))\cdot h'(x)$

$f(x)=(3x^2-5)^7$ $\qquad$ $f'(x)=7(3x^2-5)^6\cdot(6x)$

$h(x)=3x^2-5 \to h'(x)=6x$

$g(x)=x^7 \to g'(x)=7x^6$

9. IF $f(x)=\sin x$, THEN $f'(x)=\cos x$.

$\hookrightarrow f'(x)=\lim\limits_{h\to 0}\dfrac{\sin(x+h)-\sin x}{h}\ \ldots\ \cos x$

HW ↑ FILL IN DOTS.

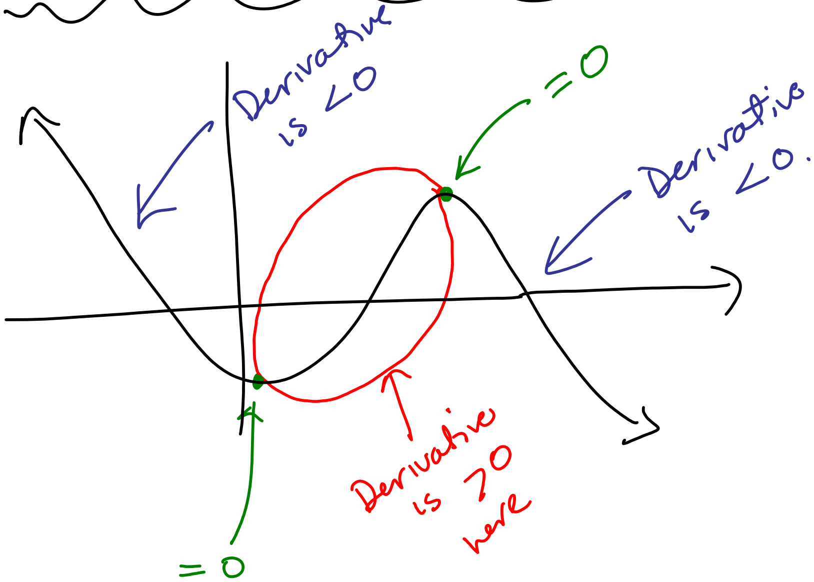
IF $f(x)=\cos x$, THEN $f'(x)=-\sin x$

IF $f(x)=\tan x$, THEN $f'(x)=\sec^2 x$

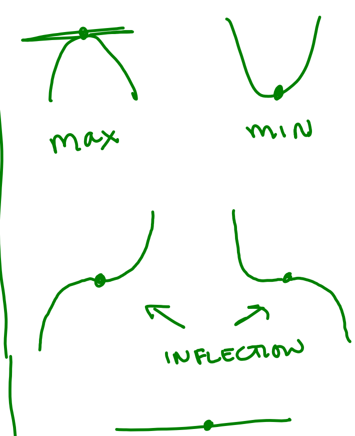
IF $f(x) = \sec x$, THEN $f'(x) = \sec x \tan x$

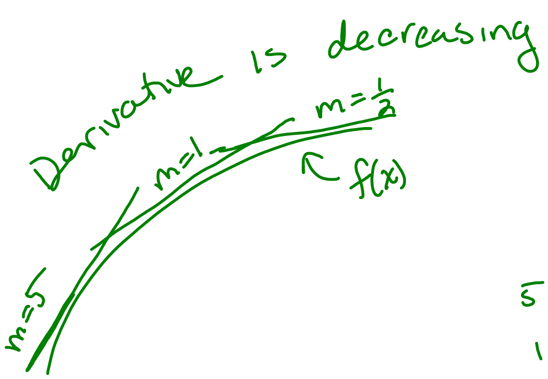
IF $f(x) = \csc x$, THEN $f'(x) = -\csc x \cot x$

IF $f(x) = \cot x$, THEN $f'(x) = -\csc^2 x$

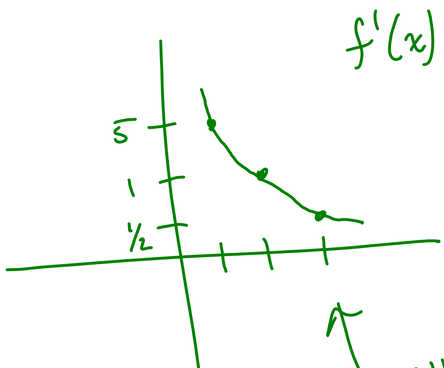


IF $f'(a) = 0$

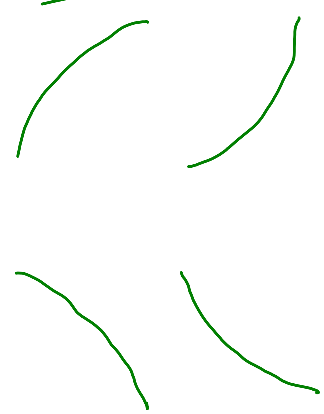




Derivative of the derivative must be negative



$f''(x) < 0$ since $f'(x)$ is decreasing



IF

$f(x) > 0$	graph is above x-axis
< 0	" " below x-axis
$f'(x) > 0$	graph is climbing
< 0	" " falling
$f''(x) > 0$	graph is concave up
< 0	graph is concave down

10. IF $y = e^x$, $y' = e^x$.

IF $y = b^x$, $y' = \ln b \cdot b^x$

IF $y = \ln x$, $y' = \frac{1}{x}$

IF $y = \log_b x$, $y' = \frac{1}{\ln b} \cdot \frac{1}{x}$