CALCULUS

- **LIMITS** & CONTINUOUS at $x = a$
  1. $f(a)$ EXISTS
  2. $\lim_{x \to a} f(x)$ EXISTS (NOT ∞, $-\infty$)
  3. 1 = 2.

ALL FUNCTS WE TALK ABOUT PIECEWISE FCTNS
ARE CONT. ON DOMAINS.

- **DERIVATIVES**

$$f(x) = \frac{\sqrt{x} - 2x^3}{x} = \frac{\sqrt{x}}{x} - \frac{2x^3}{x} = x^{-1/2} - 2x^2$$

$$f'(x) = -\frac{1}{2}x^{-3/2} - 4x.$$

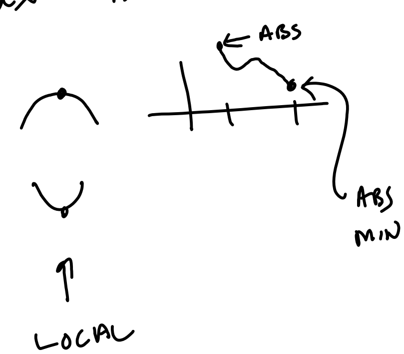
SLOPE OF TANGENT = RATE of CHANGE

NOTATION $\quad y = f(x)$: $\quad y' = f'(x) = \frac{dy}{dx} = \frac{df}{dx} = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

GRAPHING

MAXIMA — LOCAL / ABS.
REL / EXT.

MINIMA — REL / EXT
LOCAL / ABS.



ANTIDERIVATIVES

RULES, 1 TECHNIQUE — U-SUBS.

$$\int \cos x \, e^{\sin x} \, dx$$

$u = \sin x$

$\dfrac{du}{dx} = \cos x$

$dx = \dfrac{du}{\cos x}$

$$= \int \cancel{\cos x}\, e^{u} \, \frac{du}{\cancel{\cos x}}$$

$$= \int e^{u} \, du = e^{u} + C$$

$$= e^{\sin u} + C.$$

1. CHOOSE u. (Never choose $u = x$.)
2. COMPUTE $\dfrac{du}{dx}$
3. "SOLVE" FOR dx
4. SUBSTITUTE IN & SIMPLIFY (ONLY u's — NO x's)
5. INTEGRATE
6. BACK SUBS.

TIPS ABOUT PICKING u:

1. $\int \sim e^{\sim} \, dx$ — u
2. $\int \sim (\sim)^{P} \, dx$ — u
3. IF $\ln$ shows up ANYWHERE, THAT'S u.

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ex

$$\int \frac{1}{x \ln x} \, dx = \int \frac{1}{\cancel{x} \cdot u} \, \cancel{x} \, du = \int \frac{1}{u} \, du = \ln|u| + C$$

$$u = \ln x \quad \frac{du}{dx} = \frac{1}{x} \rightarrow dx = x \, du \quad = \ln|\ln x| + C$$

$$\int \ln x \, dx = ?$$ ← INTEGRATION BY PARTS!

Recall the prod. rule

$$(u \cdot v)' = u' \cdot v + v' \cdot u$$

$$u \cdot v + C = \int u' \cdot v + v' \cdot u \, dx$$

$$u \cdot v + C = \int u' \cdot v \, dx + \int v' u \, dx$$

$$uv - \int v \cdot u' \, dx = \int v' u \, dx$$ ← I.B.P.

$$\int \underbrace{\ln x}_{u} \, \underbrace{dx}_{v' \cdot dx} = (\ln x) \cdot x - \int x \cdot \frac{1}{x} \, dx$$

$u' = \frac{1}{x}$ $\quad v' = 1$

$v = x$

$$= (\ln x)(x) - \int dx$$

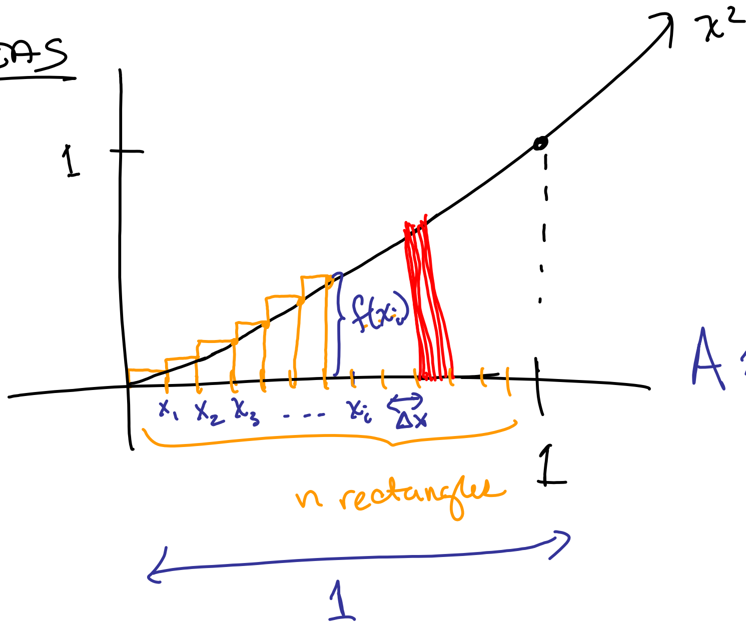
$$= (\ln x)(x) - x + C$$

$$\int u v' dx = uv - \int v u' dx$$

$\quad dv \qquad\qquad\qquad du$

$$\int u \, dv = uv - \int v \, du$$ — FORMULA SHEET VERSION.

AREAS



AREA OF i^{th} rectangle

$$A \approx \sum_{i=1}^{n} f(x_i) \cdot \frac{1}{n}$$

$$= f(x_1) \cdot \frac{1}{n} + f(x_2) \frac{1}{n} + \dots$$

$$\dots f(x_n) \cdot \frac{1}{n}$$

$$A = \lim_{n \to \infty} \sum_{i=1}^{n} f(x_i) \cdot \Delta x = \underbrace{\int_0^1 f(x)\, dx}_{\text{DEFINITE INTEGRAL}}$$

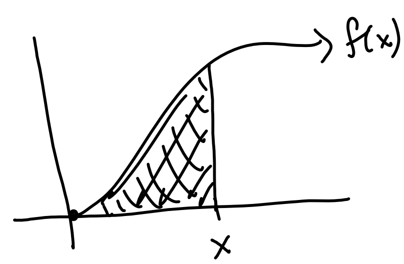
THE FUNDAMENTAL THEOREM OF CALCULUS.

IF $F(x)$ IS ANY ANTIDERIVATIVE OF $f(x)$ THEN

① $\displaystyle\int_a^b f(x)\,dx = F(b) - F(a)$.

② LET $g(x) = \displaystyle\int_a^{h(x)} f(t)\,dt \left(= F(x) - F(0) \right)$

THEN $g'(x) = f(h(x)) \cdot h'(x)$



$$g(x) = \int_0^x f(t)\, d(t)$$

ex

$$g(x) = \int_a^x 3t^3 - \sin t \, dt$$

FIND $g'(x) = 3x^3 - \sin x$

ex

$$g(x) = \int_a^{\cos x} 3t^2 - e^t \, dt$$

inside

$$g'(x) = \left(3(\cos x)^2 - e^{\cos x}\right)(-\sin x)$$