DATA ANALYSIS & STAT PROBLEMS

PROOFS

IF

① GIVEN THAT r_1 AND r_2 ARE ROOTS OF $f(x) = x^2 + bx + c$,
SHOW THAT $r_1 + r_2 = -b$ AND $r_1 r_2 = c$.

THEN

proof IF r_1 AND r_2 ARE THE ROOTS OF $x^2 + bx + c$,
THEN $x^2 + bx + c$ FACTORS AS $(x - r_1)(x - r_2)$.

$$\text{so } x^2 + bx + c = x^2 - r_1 x - r_2 x + r_1 r_2$$

$$\cancel{x^2} + bx + c = \cancel{x^2} + (-r_1 - r_2)x + r_1 r_2, \text{ so}$$

$$bx + c = (-r_1 - r_2)x + r_1 r_2$$

$$bx - (-r_1 - r_2)x + c - r_1 r_2 = 0$$

$$bx + (r_1 + r_2)x + (c - r_1 r_2) = 0$$

$$\underbrace{(b + r_1 + r_2)}_{=0}x + \underbrace{(c - r_1 r_2)}_{=0} = 0$$

so $r_1 + r_2 = -b$ AND $c = r_1 r_2$.

since the CONSTANT TERMS MUST BE EQUAL $c = r_1 r_2$, AND SINCE COEFFS OF x MUST BE EQUAL, $b = -r_1 - r_2$ ie. $-b = r_1 + r_2$.

---

2. IF $f(x)$ and $g(x)$ HAVE AS THEIR DOMAINS ALL REAL NUMBERS, THEN THE DOMAIN OF $f(g(x))$ IS ALL REAL NUMBERS.

WTS ANY REAL NUMBER CAN BE PLUGGED INTO $f(g(x))$.

ANY NUMBER, a, CAN BE THE INPUT OF g, AND g WILL GIVE AS OUTPUT A REAL NUMBER, $g(a)$. SINCE ANY REAL NUMBER CAN BE INPUT TO f, IN PARTICULAR, $g(a)$ CAN BE, $f(g(a))$ WILL YIELD A REAL OUTPUT.

SO a IS IN DOMAIN of $f(g(x))$.

8

$$1, \frac{4}{3}, \frac{9}{5}, \frac{16}{7}, \frac{25}{9}, \ldots$$

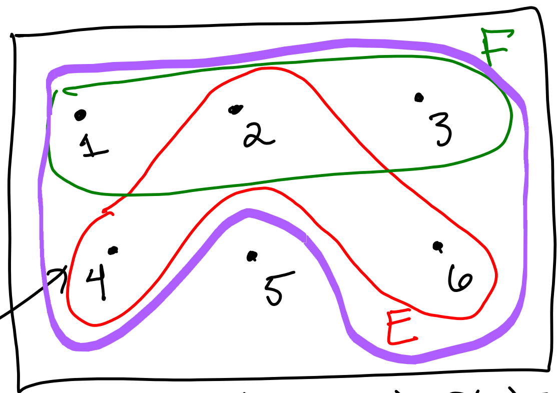
$$\frac{1^2}{1} \quad \frac{2^2}{2+1} \quad \frac{3^2}{4+1} \quad \frac{4^2}{6+1} \quad \frac{5^2}{8+1}$$

$$2-1 \quad 4-1 \quad 6-1 \quad 8-1 \quad 10-1$$

$$\left\{\frac{(n+1)^2}{2n+1}\right\}_{n=0}$$

$$\left\{\frac{n^2}{2n-1}\right\}_{n=1}$$

PROBABILITY.



OUTCOME

SAMPLE SPACE $= S$

COLLECTION OF ALL POSSIBLE OUTCOMES OF THE EXPERIMENT

$P(E) = P(2) + P(4) + P(6) = \frac{1}{6} + \frac{1}{6} + \frac{1}{6} = \frac{1}{2}$

$P(S) = 1$

$P(E) = P(O_1) + P(O_2) + \dots$

E – THE EVENT OF GETTING AN EVEN NUMBER

AN EVENT IS ANY COLLECTION OF OUTCOMES

EQUALLY LIKELY OUTCOMES $\Rightarrow P(O) = \dfrac{1}{\#\text{ of outcomes}}$

EVENTS E & F (ABOVE) ARE NOT MUTUALLY EXCLUSIVE.

(TO BE MUTUALLY EXCLUSIVE, THEY'D HAVE TO SATISFY $E \cap F = \emptyset$.)

PROBABILITY OF E OR F OCCURING MEANS E OR F OR BOTH.

$P(E) = P(2) + P(4) + P(6)$

$P(F) = P(1) + P(2) + P(3)$

$P(E \cup F) = P(1) + P(2) + P(3) + P(4) + P(6)$

$\parallel$

$P(E \cup F) = P(E) + P(F) - P(E \cap F)$

$$\underbrace{P(F \text{ DOESN'T HAPPEN})}_{P(\overline{F})} = 1 - P(F)$$

CONDITIONAL PROBABILITY

PROBABILITY THAT ONE EVENT, E, GIVEN THAT ANOTHER EVENT, F, IS KNOWN TO HAVE HAPPENED.

$$P(E|F) = \frac{P(E \cap F)}{P(F)}$$

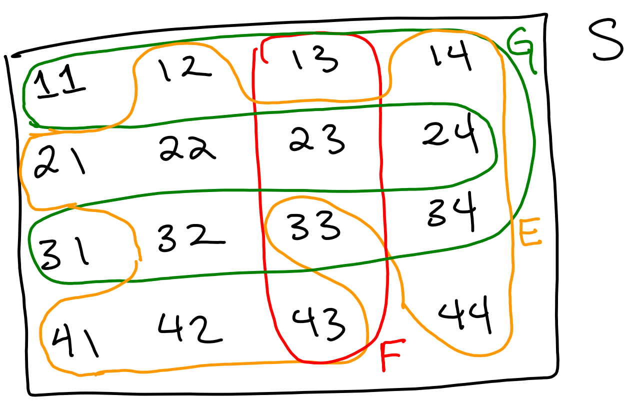
given that

$$P(E \cap F) = P(F) \cdot P(E|F)$$ ← MULTIPLICATION RULE.

INDEPENDENT EVENTS

E AND F ARE INDEPENDENT IF E OCCURING DOES NOT AFFECT WHETHER OR NOT F WILL OCCUR

$P(e) = \frac{1}{16}$



$E = \{\text{PRODUCT OF THE ROLLS IS EVEN}\}$

$F = \{\text{ROLL 3 ON 2nd ROLL}\}$

$G = \{\text{ODD ON FIRST ROLL}\}$

$$P(E) = 12 \cdot P(o) = \frac{12}{16} = \frac{3}{4}$$

$$P(F) = \frac{4}{16} = \frac{1}{4} \qquad P(E|F) = \frac{P(E \cap F)}{P(F)}$$

$$P(G) = \frac{8}{16} = \frac{1}{2} \qquad = \frac{2/16}{1/4} = \frac{2}{4} = \frac{1}{2}$$

$$P(F \cap G) = P(13) + P(33) = \frac{2}{16} = P(F) \cdot P(G) = \frac{1}{4} \cdot \frac{1}{2} = \frac{1}{8}$$

ODDS OF AN EVENT HAPPENING

\# OF WAYS IT CAN HAPPEN : # OF WAYS IT DOESN'T HAPPEN

EXPECTED VALUE

$$E(X) = \underbrace{P(o_1) \cdot o_1 + P(o_2) \cdot o_2 + \ldots}_{\text{over whole sample space.}}$$

RANDOM VARIABLE

WHAT YOU'RE MEASURING (NUMBER VALUE)

$$E(X) = \frac{1}{6} \cdot 1 + \frac{1}{6} \cdot 2 + \frac{1}{6} \cdot 3 + \frac{1}{6} \cdot 4 + \frac{1}{6} \cdot 5 + \frac{1}{6} \cdot 6$$

$$= \frac{1}{6}(21) = \frac{21}{6} = 3.5$$

GAME FLIP AN UNFAIR COIN. $\left(P(H) = \frac{1}{4} \quad P(T) = \frac{3}{4}\right)$

IF H, PLAYER A MUST GIVE PLAYER B 3.

IF T, PLAYER B MUST GIVE PLAYER A 2.

WHAT CAN PLAYER A EXPECT TO WIN ON AN AVERAGE HAND.

$$E(X_A) = P(H)\cdot(-3) + P(T)\cdot(2)$$

$$= \frac{1}{4}\cdot -3 + \frac{3}{4}\cdot 2 = -\frac{3}{4} + \frac{6}{4} = \frac{3}{4} = 0.75$$

$$E(X_B) = \frac{1}{4}(3) + \left(\frac{3}{4}\right)(-2) = \frac{3}{4} - \frac{6}{4} = -0.75$$

THEORETICAL PROBABILITY – "TRUE" PROB.

EMPERICAL PROBABILITY – EXPERIMENTAL

PROB (MULTIPLE CHOICE)

\#2 $P(H) = 2P(T)$ $\quad P(H) + P(T) = 1$

$P(T) = \frac{1}{3}$, $P(H) = \frac{2}{3}$

$$P(E) = P(o_1) + P(o_2) + P(o_3)$$

$$= \frac{1}{3}\cdot\frac{1}{3}\cdot\frac{2}{3} + \frac{2}{27} + \frac{2}{27} = \frac{2}{9}$$

HHH HHT HTH

THH

E

TTH 1

THT 2

HTT 3

TTT

#3

	P	N
S	25	5
N	15	155

$$P(p|s) = \frac{P(p \cap s)}{P(s)}$$

$$= \frac{25/200}{30/200} = \frac{25}{30}$$

$$= \frac{5}{6}$$

#4

111 112 121

247, 151, 26

$$\frac{}{365^3}$$

$\underline{*}\ \underline{*}$, $\underline{?}$

(1, 1) (2,1)

(2, 1) (2,2)

(3, 3)

$365 \cdot 365$

$\underline{*}\ \underline{?}\ \underline{*}$

$\underline{?}\ \underline{*}\ \underline{*}$

$$\frac{3 \cdot (365)^2}{(365)^3} = \frac{3}{365}$$

$$\underset{365}{\uparrow}\ \underset{365}{\uparrow}\ \underset{365}{\uparrow} = 365^3$$

$$1 - \underbrace{P\left(\begin{matrix}\text{DISTINCT}\\ \text{B-DAYS}\end{matrix}\right)} = P\left(\begin{matrix}\text{AT LEAST}\\ \text{2 MATCH}\end{matrix}\right)$$

$$\underset{365}{\uparrow} \cdot \underset{364}{\uparrow} \cdot \underset{363}{\uparrow}$$

$$\frac{365 \cdot 364 \cdot 363}{365^3}$$