MATRIX ALGEBRA !

4 COLUMNS

3 ROWS $\left\{ \begin{bmatrix} 2 & 3 & 1 & 5 \\ 7 & 6 & 5 & -2 \\ 0 & -9 & \frac{1}{3} & 7 \end{bmatrix} \right.$ ← MATRIX

3×4

ROWS x COLUMNS

IF ROWS = COLS ⇒ SQUARE MATRIX

$\begin{bmatrix} 1 & 6 \\ 0 & 1 \end{bmatrix}$ 2×2 ↗

$[3]$ 1×1

1

$3 \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ 3×1 → COLUMN VECTOR (ONLY 1 COL.)

$[x \quad y \quad z]$ 1×3 → ROW VECTOR

$\begin{bmatrix} 5 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & -3 \end{bmatrix}$

A SQUARE MATRIX IS DIAGONAL IF ALL ENTRIES NOT ON THE DIAG ARE ZERO

$$I_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$ SQUARE MATRIX WITH ALL 1's ON DIAG & ZEROS OFF DIAG IS IDENTITY MATRIX.

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I_2$$

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$ ALL ZEROS $\Rightarrow$ THE ZERO MATRIX

TRANSPOSE OF A MATRIX:

$$\begin{bmatrix} 2 & 1 & 5 \\ 3 & 2 & 7 \end{bmatrix}^T = \begin{bmatrix} 2 & 3 \\ 1 & 2 \\ 5 & 7 \end{bmatrix}$$ $a_{ij} \rightarrow a_{ji}$

a_{ij} ← entry in the ith row & j col eg $a_{23} = 7$ $a_{21} = 3$

$a_{12} = 1$

IF FOR MATRIX M, $M^T = M$,

WE CALL M symmetric.

ex $\begin{bmatrix} 2 & 5 \\ 5 & 3 \end{bmatrix}^T = \begin{bmatrix} 2 & 5 \\ 5 & 3 \end{bmatrix}$

ADD - CAN ONLY DO THIS IF DIMENSIONS MATCH.

$$\begin{bmatrix} 2 & 3 & 0 \\ 0 & 1 & 2 \end{bmatrix} + \begin{bmatrix} 1 & 5 & -1 \\ 0 & -6 & 0 \end{bmatrix} = \begin{bmatrix} 2+1 & 3+5 & 0+-1 \\ 0 & -5 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & 8 & -1 \\ 0 & -5 & 2 \end{bmatrix}$$

SUBTRACTION - SAME IDEA.

$$\begin{bmatrix} 2 & 3 \\ 0 & 5 \end{bmatrix} + \begin{bmatrix} -2 & -3 \\ 0 & -5 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\left(\begin{bmatrix} x & 3 \\ y-1 & 4 \end{bmatrix} = \begin{bmatrix} 4 & z \\ 3 & 4 \end{bmatrix} \right.$$ CORRESPONDING ENTRIES of = MATRICES ARE =.

SCALAR MULTIPLICATION

$$\frac{7}{3} \cdot \begin{bmatrix} 1 & 2 & 5 \\ 3 & 4 & 6 \end{bmatrix} = \begin{bmatrix} 7/3 & 14/3 & 35/3 \\ 7 & 28/3 & 42/3 \end{bmatrix}$$

ex $3X + I_3 = \begin{bmatrix} 1 & 0 & 1 \\ 2 & -1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$

$$3 \cdot \begin{bmatrix} - & - & - \\ - & - & - \\ - & - & - \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1 \\ 2 & -1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$$

$$3 \cdot \begin{bmatrix} 0 & 0 & 1/3 \\ 2/3 & -2/3 & 0 \\ 0 & 1/3 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ 2 & -2 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

MATRIX MULTIPLICATION

MUST HAVE $m = r$ IN ORDER TO CALCULATE A PRODUCT

$n \times m \quad r \times s$

$$M_1 \cdot M_2 = M_3 \quad (n \times s)$$

$3 \times 2 \quad 2 \times 2$

$$\begin{bmatrix} 3 & 0 \\ 2 & -1 \\ 1 & 4 \end{bmatrix} \cdot \begin{bmatrix} 2 & 1 \\ 5 & 6 \end{bmatrix} = \begin{bmatrix} 3 \cdot 2 + 0 \cdot 5 & 3 \cdot 1 + 0 \cdot 6 \\ -1 & -4 \\ 22 & 25 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 7 & 10 \\ 3 & 4 \end{bmatrix}$$

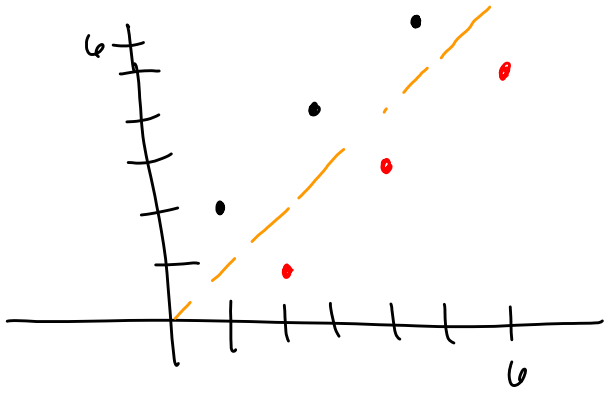
$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 4 \\ 3 & 10 \end{bmatrix}$$

MATRIX MULT. IS NOT COMMUTATIVE

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 3 & 5 \\ 2 & 4 & 6 \end{bmatrix} = \begin{bmatrix} 1 & 3 & 5 \\ 2 & 4 & 6 \end{bmatrix}$$

$2 \times 2 \qquad 2 \times 3$

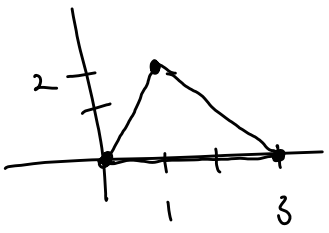
$$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 3 & 5 \\ 2 & 4 & 6 \end{bmatrix} = \begin{bmatrix} 2 & 4 & 6 \\ 1 & 3 & 5 \end{bmatrix}$$



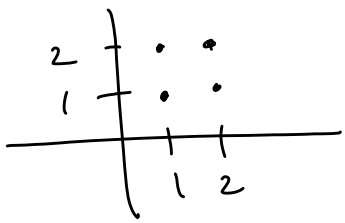
$$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} 1 & 3 & 5 \\ 2 & 4 & 6 \end{bmatrix} = \begin{bmatrix} 1 & 3 & 5 \\ -2 & -4 & -6 \end{bmatrix}$$

$$\begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} 0 & 1 & 3 \\ 0 & 2 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 3 & 9 \\ 0 & 6 & 0 \end{bmatrix}$$

$$k \cdot M = \begin{bmatrix} k & 0 & 0 \\ 0 & k & 0 \\ 0 & 0 & k \end{bmatrix} \cdot M$$

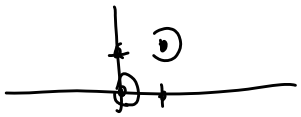


$$\begin{bmatrix} 2 & 2 & 2 \\ 3 & 3 & 3 \end{bmatrix} + \begin{bmatrix} 0 & 1 & 3 \\ 0 & 2 & 0 \end{bmatrix}$$



$$\begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}\begin{bmatrix} 1 & 2 & 1 & 2 \\ 1 & 2 & 2 & 1 \end{bmatrix} =$$

$(3,3)\ (6,6)$



NO DIVIDING?

$A \cdot x = b$ ←

$$2x - 3y = 7$$
$$5x + y = 6$$

$$\begin{bmatrix} 2 & -3 \\ 5 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 7 \\ 6 \end{bmatrix}$$

A: 2×2 — 2×1: x — 2×1: b

$$Ax = b$$

PLAN: ① FIND A^{-1}

② MULT. ON BOTH A^{-1}

$$A^{-1}Ax = A^{-1}b$$

$A^{-1}A = I$

$$x = A^{-1}b$$

INVERSE OF 2×2 MATRICES

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$$A^{-1} = \frac{1}{\det A} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}, \text{ where } \det(A) = ad - bc$$

↑ the determinant of A

$$A = \begin{bmatrix} 2 & -3 \\ 5 & 1 \end{bmatrix}$$

$$\det A = 2 \cdot 1 - ((-3) \cdot 5) = 2 + 15 = 17$$

$$A^{-1} = \frac{1}{17} \begin{bmatrix} 1 & 3 \\ -5 & 2 \end{bmatrix} = \begin{bmatrix} 1/17 & 3/17 \\ -5/17 & 2/17 \end{bmatrix}$$

$$\underbrace{A^{-1} A}_{I} \vec{x} = A^{-1} \begin{bmatrix} 7 \\ 6 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \vec{x} = \begin{bmatrix} 1/17 & 3/17 \\ -5/17 & 2/17 \end{bmatrix} \begin{bmatrix} 7 \\ 6 \end{bmatrix} = \begin{bmatrix} 25/17 \\ -23/17 \end{bmatrix}$$

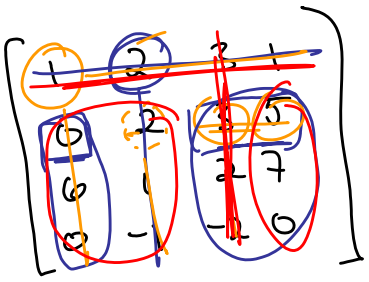
$$B = \begin{bmatrix} 2 & 6 \\ 1 & 3 \end{bmatrix} \qquad \det B = 0 \Rightarrow \text{NO INVERSE!}$$

Ex

$$\begin{bmatrix} 2 & 3 & 1 \\ 4 & 0 & 5 \\ -1 & 2 & 1 \end{bmatrix}$$

$+2\cdot(0-10)-3(4+5)+1(8-0)$

Ex



$+1(\det[\,])-2\left(0\cdot(\det[\,])-3\det[\,]+5\det[\,]\right)$

$+3(0\cdot$

Ex

$$\begin{bmatrix} 3 & 7 & 9 & 6 \\ 5 & 2 & 1 & 4 \\ 8 & 3 & 9 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$-0\cdot(\quad)+0(\quad)-1(\quad)+0(\quad)$