MC. PRACTICE EXAM 3

4

$$y = 25x^2 - kx + 16$$

$$= (5x-4)^2 = 25x^2 - 40x + 16$$

5

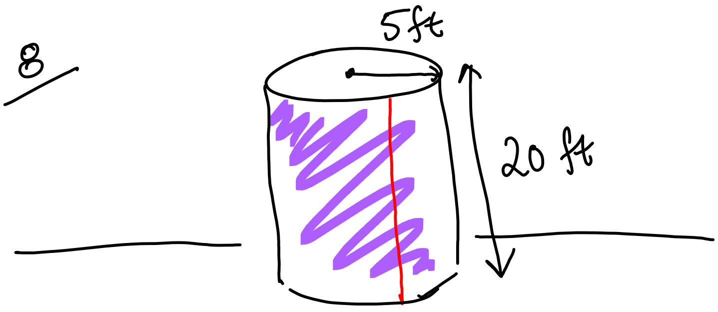
DIST FROM (x_1, y_1) TO $Ax + By + C = 0$

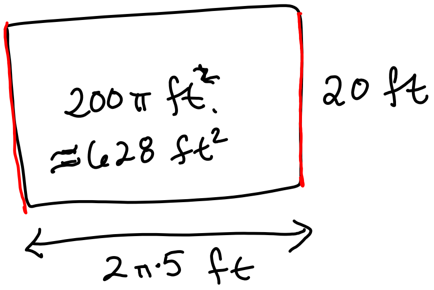
$$d = \frac{|Ax_1 + By_1 + C|}{\sqrt{A^2+B^2}}$$

$$\underset{A}{\underset{\uparrow}{4}}x + \underset{B}{\underset{\uparrow}{3}}y \underbrace{-5}_{C} = 0$$

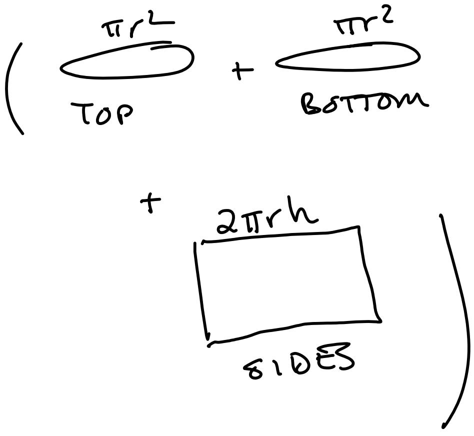
$= 0.8$

8





SURFACE AREA OF CYLINDER



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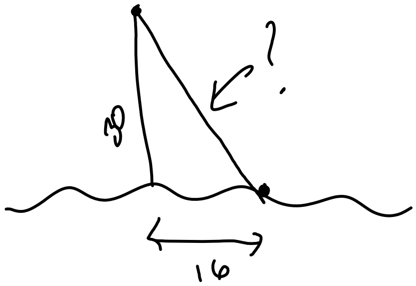
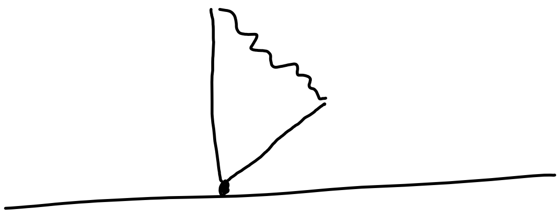
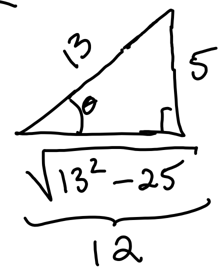
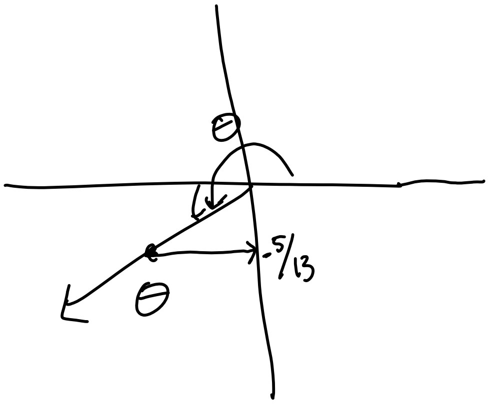




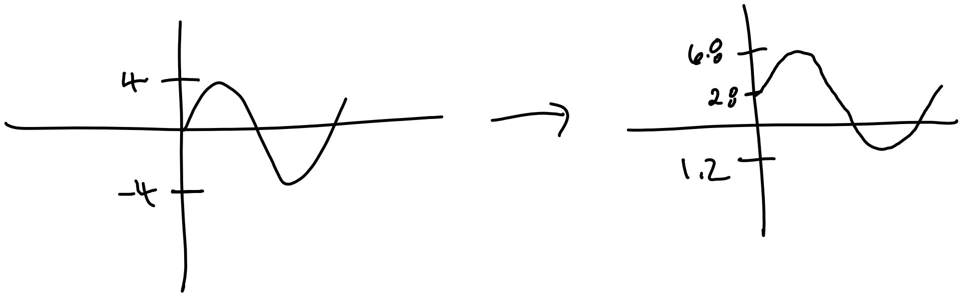
11.

$\tan\theta = \frac{5}{12}$



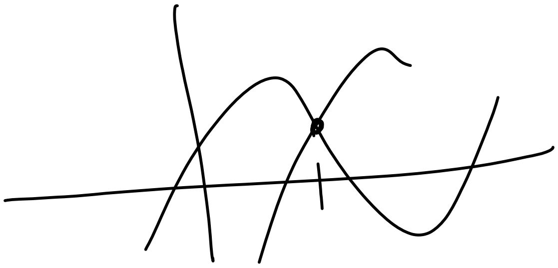


13. $y = 4\sin x + 2.8$



$$\frac{\tan\theta + \cot\theta}{\sec\theta \csc\theta} = \frac{\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}}{\frac{1}{\cos\theta}\cdot\frac{1}{\sin\theta}} = \frac{\frac{\sin^2\theta + \cos^2\theta}{\cancel{\sin\theta\cos\theta}}}{\frac{1}{\cancel{\sin\theta\cos\theta}}}$$

$$= \sin^2\theta + \cos^2\theta \; (=1)$$



1c "conformable for mult"

means you can do it

For $n\times n$ MATRICES

$A\,c\,B$ iff $\overset{n\times m}{A}\;\overset{m\times l}{B}$ $\quad \overset{=}{m}$

$n\times n \quad n\times n$

$n\times m$

$\overset{3\times 2}{A} \cdot \overset{2\times 5}{B}$

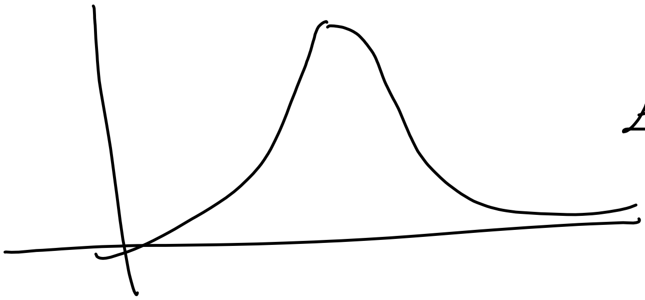
$\overset{2\times 5}{B} \cdot \overset{5\times 1}{C}$

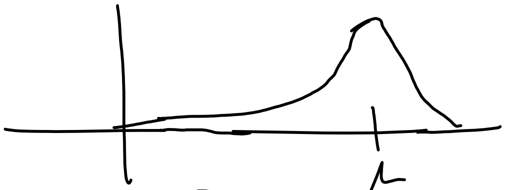
$\Rightarrow A\cdot C$?

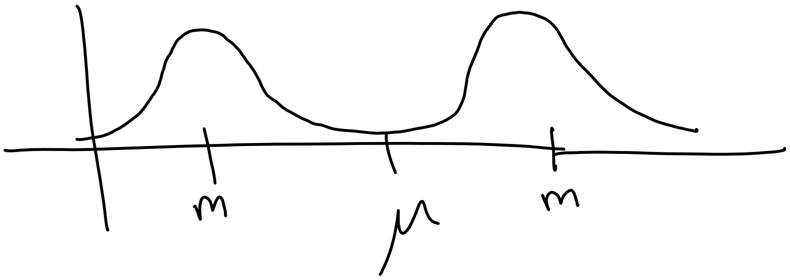
$A\,c\,A$

$A\,c\,B$ THEN $B\,c\,A$

$A\,c\,B$ AND $B\,c\,D \rightarrow A\,c\,D$







18/

$$\frac{\text{AREA of little circ}}{\cancel{\text{AREA}}\text{ of Big circ}} = \frac{\pi\left(\frac{d}{2}\right)^2}{\pi\left(\frac{D}{2}\right)^2} = \frac{d^2}{D^2} = \frac{\left(\frac{1}{4}D\right)^2}{D^2}$$

$$= \frac{1}{16}$$

40.60

20/ 40.6mL

a) 40.595 — 40.605

b) 40.55 — 40.65

21

$$\log_{10}\left(\frac{x^3}{20}\right) = \log_{10} x^3 - \log_{10} 20$$

$$= 3\log_{10} x - \log_{10} 20$$

$\log\left(\frac{a}{b}\right) = \log a - \log b$

$\log(a \cdot b) = \log a + \log b$

$\log(a^b) = b\log a$

$$= 3\log_{10} x - \left(\log_{10} 2 \cdot 10\right) =$$

$$= 3\log_{10} x - \left(\log_{10} 2 + \log_{10} 10\right)$$

$$3\log_{10} x - \log_{10} 2 - 1$$

$(\log a)^b \neq$ ← DON'T MAKE THIS ERROR!

$\neq \log a^b = \log(a^b)$

22 A IS MATRIX: $\det A = 0$

$\iff$ A is singular

$\iff$ A IS NON-INVERTIBLE

↳ CAN'T FIND AN INVERSE

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} r \\ s \end{bmatrix}$$

$ax + by = r$

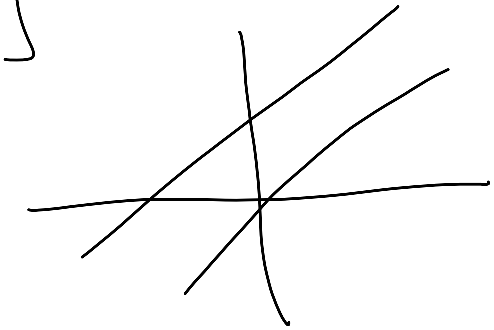
$cx + dy = s$

$ad = bc$

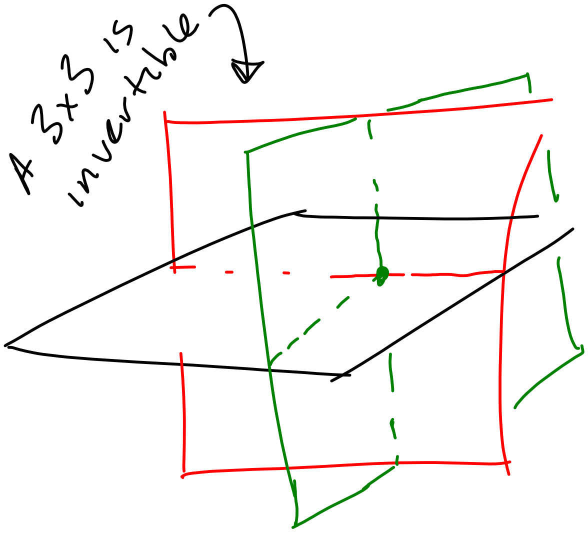
$$\begin{bmatrix} 2 & 1 \\ 4 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} r \\ s \end{bmatrix}$$

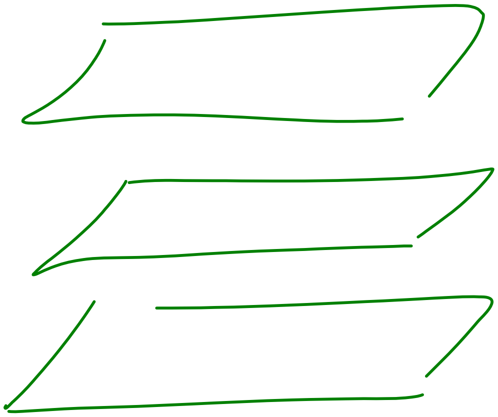
$-\frac{A}{B}$

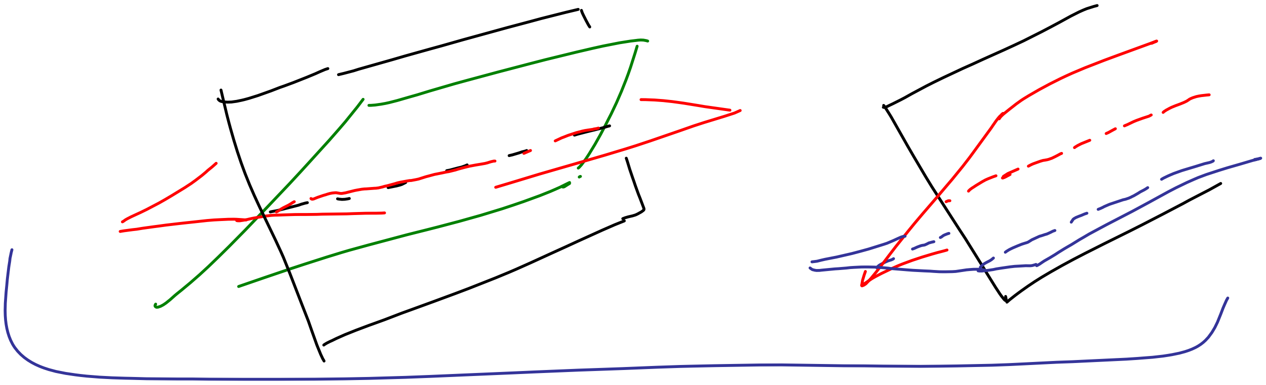
$$2x + y = 3$$
$$4x + 2y = 7$$



$$\rightarrow 2x + 3y - z = 5 \; \}$$







23

$S(x) = 5000 \ln x + 1500$

$k

$20000 = 5000 \ln x + 1500$

$\frac{185\cancel{00}}{50\cancel{00}} = \ln x$

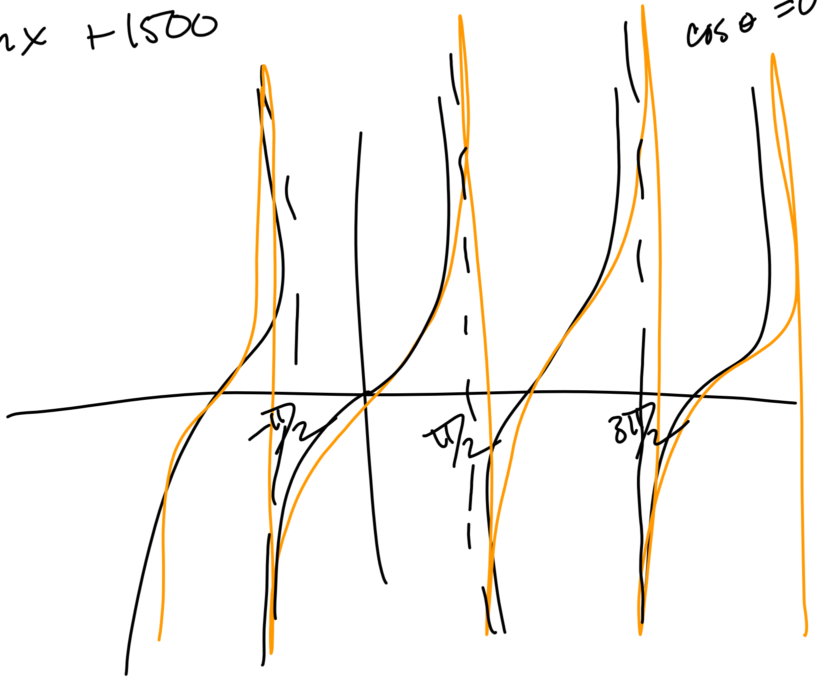
$3.7 = \ln x$

$e^{3.7} = e^{\ln x}$

$40 = x$

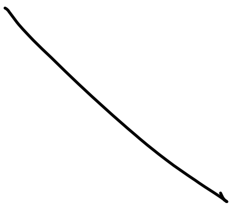
$\tan\theta = \frac{\sin\theta}{\boxed{\cos\theta}}$

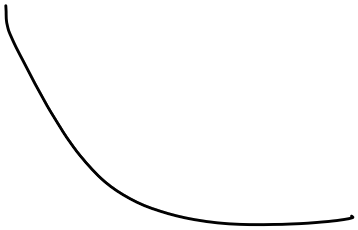
undef when $\cos\theta = 0$



25 $f'(x) = 64x^{-5}$

$f'(2) = \frac{2^6}{2^5} = 2$





29

$$m = \frac{gr^2}{kP} = \frac{\left(\frac{m}{s^2}\right)(m)^2}{\frac{m^2}{kg^2}\cdot\frac{kg\cdot m}{s^2}} = \frac{\frac{\cancel{m^3}}{\cancel{s^2}}}{\frac{\cancel{kg}\,\cancel{m^3}}{kg^2\,\cancel{s^2}}} = kg$$

30

Compounded
k times
per year

$\longrightarrow$ Continuously Compounded

$$B = P\left(1+\frac{r}{k}\right)^{kt}$$

$$\lim_{k\to\infty}\left(1+\frac{r}{k}\right)^{kt} = e^{rt}$$

$$\boxed{\lim_{k\to\infty}\left(1+\frac{1}{k}\right)^{k} = e}$$

$$B = Pe^{rt}$$

years

interest rate

Balance after t years

Principle

ex $k=1$

AFTER 1 year $\quad P + rP = P(1+r)$

AFTER 2 years $\quad (P+rP) + r(P+rP)$

$$= P + rP + rP + r^2P$$

$$= P(1+2r+r^2)$$

$$= P(1+r)^2$$

\$100 AT 10%

100 + 10 = 110

110 + 10% of 110 = 110 + 11 = 121

#30 $P = 10k$, $k=1$

$t = 20$, $B = 20k$

$$20k = 10k(1+r)^{20}$$

$$2 = (1+r)^{20}$$

$$\ln 2 = \ln\left((1+r)^{20}\right)$$

$f'(x)=f(x)$

$\ln 2 = 20\ln(1+r)$

$\frac{\ln 2}{20} = \ln(1+r)$

$e^{\ln 2/20} = 1+r$

$r = e^{\ln 2/20} - 1$

32

$Q(h) = \frac{10(4h+25)}{2h+5}$

$\lim_{h\to\infty} \frac{10(4h+25)}{2h+5} = 20$

$\hookrightarrow = \lim_{h\to\infty} \frac{40h+250}{2h+5} \cdot \frac{\frac{1}{h}}{\frac{1}{h}} = \lim_{h\to\infty} \frac{40+\frac{250}{h}\to 0}{2+\frac{5}{h}\to 0} = \frac{40}{2} = 20$

33

Volume of water in tank, V.

$\frac{dV}{dt} = 30$. WHAT IS $\frac{dh}{dt}$

$V = \pi r^2 \cdot h$

$V = 25\pi \cdot h$

differentiate with respect to $\underline{\underline{t}}$.

$\frac{dV}{dt} = 25\pi \frac{dh}{dt}$

$30 = 25\pi \frac{dh}{dt}$

$\frac{30}{25\pi} = \frac{dh}{dt}$

34

$$\underbrace{\begin{bmatrix}2&5&-1\\-4&3&5\\8&10&0\end{bmatrix}}_{A}\begin{bmatrix}x\\y\\z\end{bmatrix}=\underbrace{\begin{bmatrix}5\\13\\-4\end{bmatrix}}_{B}$$

$$\begin{bmatrix}x\\y\\z\end{bmatrix}=A^{-1}\cdot B \qquad\qquad z=-1$$

35

$$\underset{\substack{\uparrow\\3}}{\underline{\qquad\qquad}}\ \cdot\ \overline{\ \ }\ \underset{\substack{\uparrow\\10}}{\underline{\quad}}\cdot\underset{\substack{\uparrow\\10}}{\underline{\quad}}\cdot\underset{\substack{\uparrow\\10}}{\underline{\quad}}\cdot\underset{\substack{\uparrow\\10}}{\underline{\quad}}$$

36

$$y=x^3-6x^2+12x-5$$

$$y'=3x^2-12x+12$$

$$y''=6x-12 \ \leftarrow \qquad \xleftarrow{\quad - \quad}\underset{2}{|}\xrightarrow{\quad + \quad}\ y''$$

37

$$x^4-3x^3+x^2+5=f(x)$$

$$\int_0^2 x^4-3x^3+x^2+5\,dx=\left(\frac{x^5}{5}-\frac{3x^4}{4}+\frac{x^3}{3}+5x\,\right|_0^2$$

$$=\left(\frac{2^5}{5}-3\cdot2^2+\frac{2^3}{3}+10\right)-0$$

etc. use your calculator

39/

$2x^2 - x < 1$

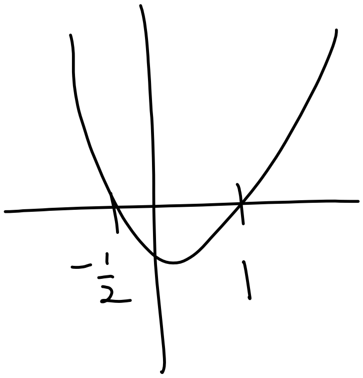
$y = \boxed{2x^2 - x - 1} < 0$

$\underbrace{(2x+1)}\underbrace{(x-1)} < 0$

① − AND +

OR

② + AND −



① OR ②

①

$2x+1<0$

&

$x-1>0$

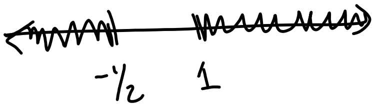
$x>1$

$2x+1<0$

$2x<-1$

$x<-1/2$

NO SOL'NS.



②

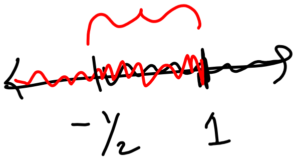
$2x+1>0$

&

$x-1<0$

$x>-1/2$

$x<1$



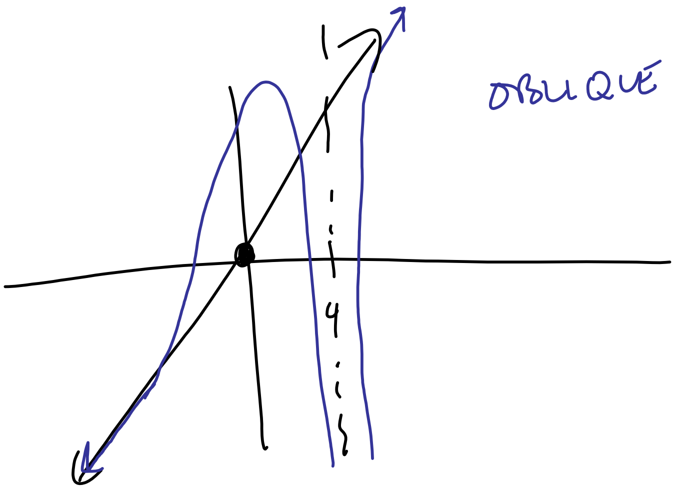
42/

$P = P_0 e^{xt}$

$500 = P_0 e^0 = P_0 \Rightarrow P_0 = 500$

$1500 = 500 e^{x5}$

$3 = e^{5x}$

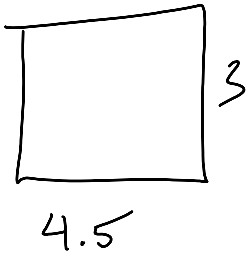


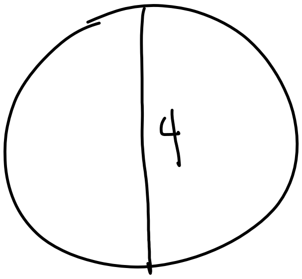
As x gets big,

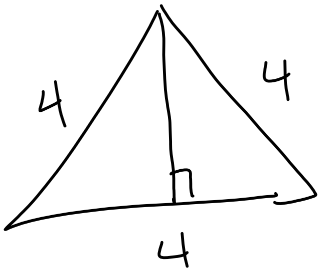
THE DOMINANT BEHAVIOR IS DICTATED BY THE leading terms $\frac{2x^3}{x^2}$

$= 2x$

45





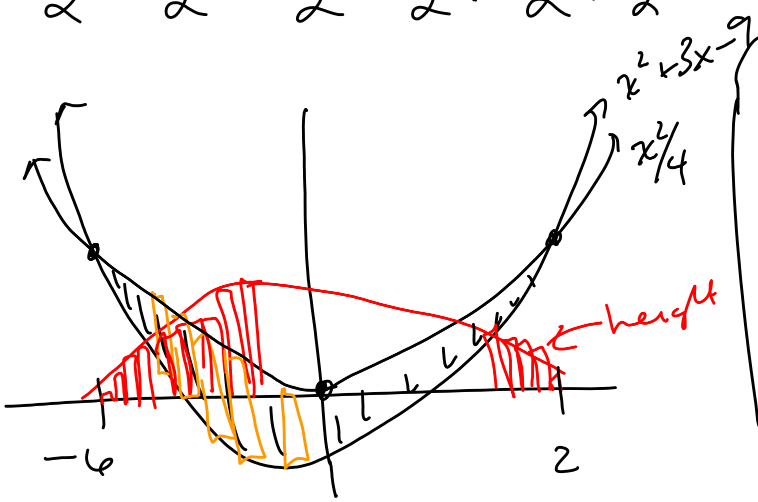


48

___ ___ ___ ___ ___ ___

2 2 · 2 · 2 · 2 · 2

49



$$x^2+3x-9 = \frac{x^2}{4}$$

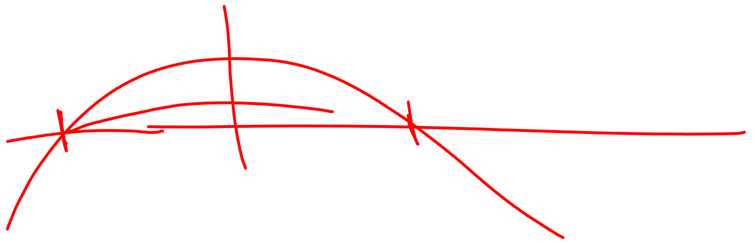
$$4x^2+12x-36 = x^2$$

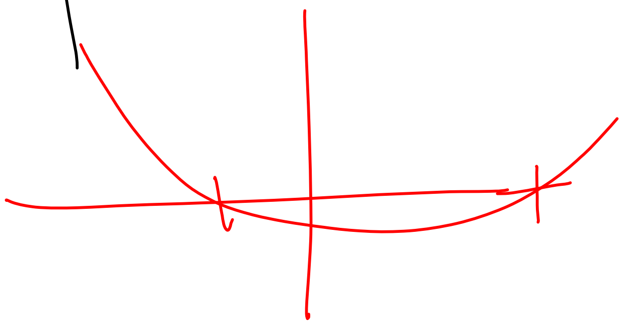
$$3x^2+12x-36 = 0$$

$$3(x^2+4x-12) = 0$$

$$\int_{-6}^{2} \underbrace{\frac{x^2}{4} - \left(x^2+3x-9\right)}_{\text{plot.}} \, dx$$

$$3(x+6)(x-2) = 0$$





$$\frac{3}{25} \cdot \frac{2}{24}$$

---

(1) (a)

$$(d \otimes a) \otimes c$$
$$= \underbrace{b}_{} \otimes c = b$$

(b) YES, BECAUSE THE $\otimes$-PRODUCT ANY TWO ELEMENTS FROM $\mathbb{X}$ IS ANOTHER EL'T FROM $\mathbb{X}$.

(c) YES, SINCE FOR ANY PAIRS $x, y \in \mathbb{X}$, $x \otimes y = y \otimes x$. THIS CAN BE CHECKED BY NOTING THAT THE TABLE IS SYMMETRIC ALONG THE DIAGONAL.

(d) C IS AN IDENTITY FOR $\otimes$ BECAUSE FOR ANY

$x \in X \quad c \otimes x = x \otimes c = x$

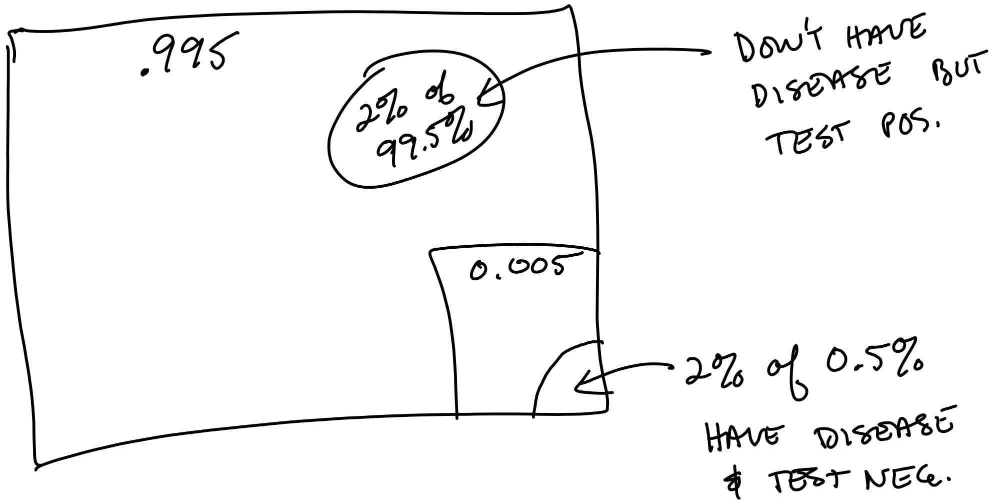
(e) INVERSE OF a is a

b is d

c is c

d is b

#3



DON'T HAVE DISEASE BUT TEST POS.

2% of 0.5% HAVE DISEASE & TEST NEG.

a) 500

b) $0.02 \times 0.995 \times 100k = 1990$

c) $\underbrace{1990}_{\text{FALSE POS.}} + \underbrace{0.98 \cdot 500}_{\text{ACTUAL POS.}} = 2480$

#2 arim. mean $\frac{a+b}{2}$

geom. mean $\sqrt{ab}$

a) 2,4 ; 1,9, ; WHEN IS $\sqrt{ab} = \frac{a+b}{2}$

↓

$$(2\sqrt{ab})^2 = (a+b)^2$$

$$4ab = a^2+2ab+b^2$$

$$0 = a^2-2ab+b^2$$

$$0 = (a-b)^2$$

So $a=b$.

b) W.T.S. $\sqrt{ab} \leq \frac{a+b}{2}$

$$2\sqrt{ab} \leq a+b$$

$$4ab \leq a^2+2ab+b^2$$

$$0 \leq a^2-2ab+b^2$$

$0 \leq (a-b)^2$ ← CLEARLY

---

proof. CLEARLY $(a-b)^2 \geq 0$ since a,b ARE REAL.

So $a^2-2ab+b^2 \geq 0$

$+4ab$ $+4ab$

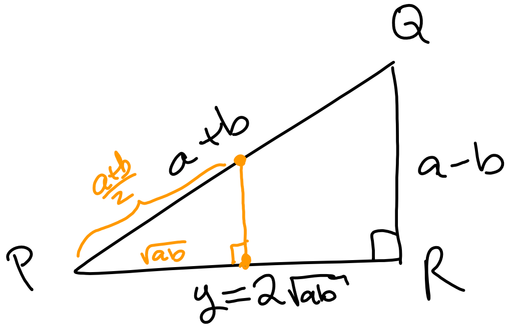
$$a^2+2ab+b^2 \geqslant 4ab$$

$(a+b)^2 \geqslant 4ab$, AND since $a,b>0$

$$a+b \geq 2\sqrt{ab}$$

$$\text{so} \quad \underset{\substack{\uparrow \\ \text{ARITH'} \\ \text{mean}}}{\frac{a+b}{2}} \geq \underset{\substack{\uparrow \\ \text{GEOMETRIC} \\ \text{mean}}}{\sqrt{ab}}$$

c)



$$(a-b)^2 + y^2 = (a+b)^2$$

$$y = \sqrt{(a+b)^2-(a-b)^2}$$

$$y = \sqrt{\cancel{a^2}+2ab+\cancel{b^2}-\cancel{a^2}+2ab-\cancel{b^2}}$$

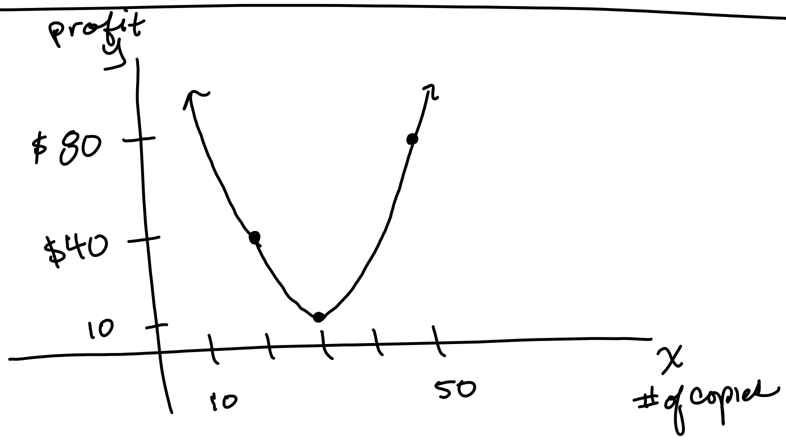
$$y = 2\sqrt{ab}$$

NOW, SINCE THE HYPOTENUSE OF A RIGHT △ IS ALWAYS THE LONGEST SIDE,

$$a+b > 2\sqrt{ab}.$$

$$\text{so} \quad \frac{a+b}{2} > \sqrt{ab}$$

#4 a)



$$y = ax^2 + bx + c$$

$$40 = 400a + 20b + c$$
$$10 = 900a + 30b + c$$
$$80 = 2500a + 50b + c$$

$$\begin{bmatrix} 40 \\ 10 \\ 80 \end{bmatrix} = \begin{bmatrix} 400 & 20 & 1 \\ 900 & 30 & 1 \\ 2500 & 50 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix}$$

$$B = A\bar{x}$$

↑
SOLVE
FOR THIS

e) $\bar{x} = A^{-1} \cdot B = \begin{bmatrix} 13/60 \\ -83/6 \\ 230 \end{bmatrix}$

$$y = \frac{13}{60}x^2 - \frac{83}{6}x + 230$$

$$e^A$$

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} = \frac{[\;]}{1} +$$

$$\sum a_n = a_0 + a_1 + a_2 + \cdots$$

$\sum$ EXISTS $\Longrightarrow \lim_{n \to \infty} a_n = 0$

$$\lim_{n \to \infty}$$

$$\lim_{n \to \infty} \frac{x^n}{n!} = 0$$

$$(\underbrace{n!})!$$

$$(5!)!$$

$$(5 \cdot 4 \cdot 3 \cdot 2 \cdot 1)! \quad (120)!$$