2.

and $\qquad\qquad\qquad$ and

$$B \cdot (C \text{ OR } M \text{ OR } (C \cdot M))$$

$$2 \cdot (3 + 4 + (3 \cdot 4))$$

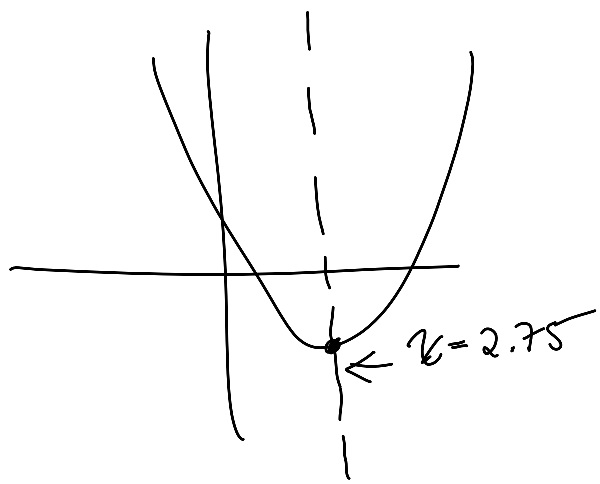
4.

$$\frac{5 \cdot 1 + 10 \cdot 2 + 15 \cdot 3 + 15 \cdot 4 + 5 \cdot 5}{5 + 10 + 15 + 15 + 5}$$

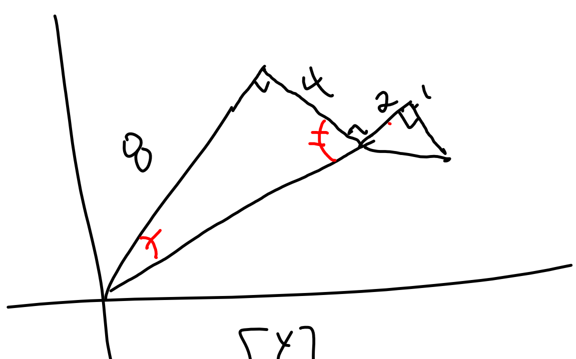
5.

$$y = ax^2 + bx + c$$

$$x_v = \frac{-b}{2a}$$



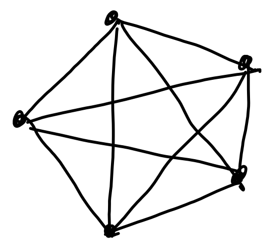

6.




9.

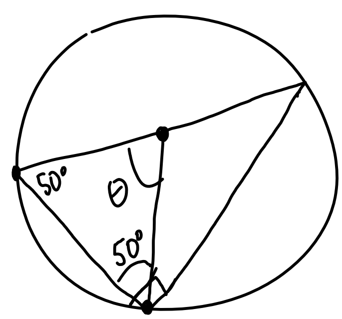
$$3 \cdot \overbrace{\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}}^{\begin{bmatrix} x \\ y \end{bmatrix}}$$

10. $\int_a^b f(x)\,dx =$ AREA ABOVE $-$ AREA BELOW $=$ "Net area"

20. 

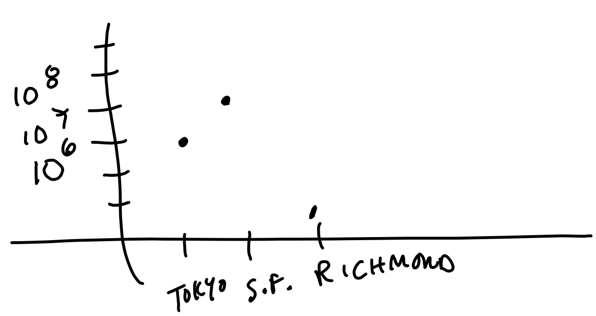
10 EDGES

$\times 2 = 20$

12. 


13. $$\lim_{t\to\infty} \frac{10000}{10+50e^{-t/2}} = 1000$$

(since $\lim_{x\to\infty} e^{-x} = 0$.)

14. 

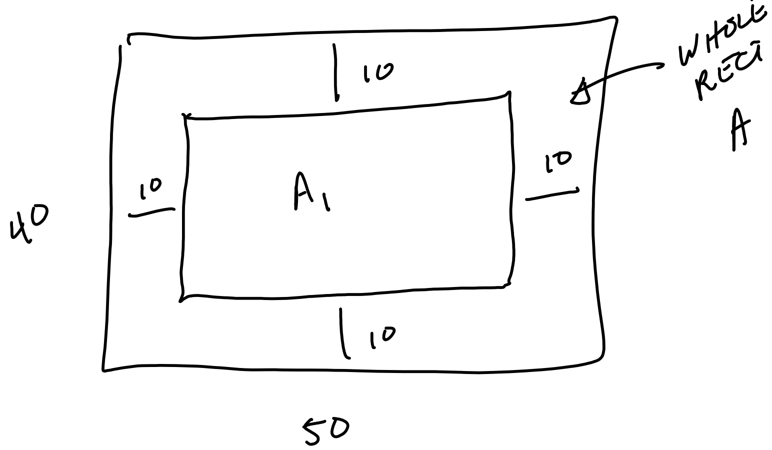

$$\frac{10^{6.8}}{10^{6.6}}$$

15. $4\pi r^2 = 1500 \leftarrow$ SOLVE FOR $r$ & PLUG IN

$r^2 = \sqrt{\left(\frac{1500}{4\pi}\right)}$

$\frac{4}{3}\pi r^3 = ?$

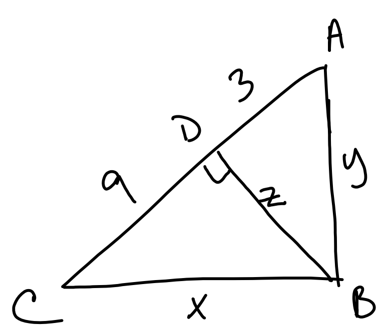
16.




$\frac{20 \times 30}{40 \times 50} = .3$

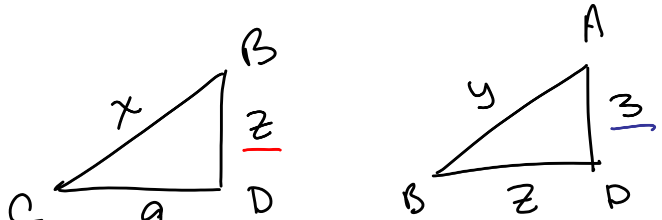
19. $h(f(g(x))) = \frac{1}{4(x^2+1)}$

22




$x + y + z = ?$

$6\sqrt{3} + 6 + 3\sqrt{3} = 6 + 9\sqrt{3}$




$27 + 81 = 108 = x^2$

$27 + 9 = y^2 = 36$

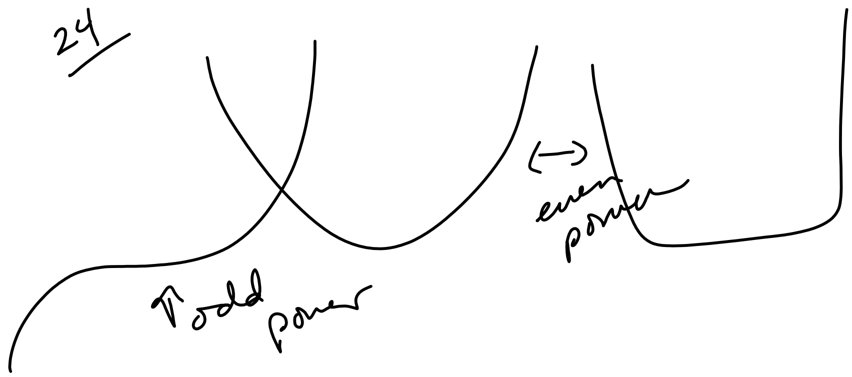
$$\frac{z}{9} = \frac{3}{z} \qquad z^2 = 27$$

$$z = 3\sqrt{3}$$

---

$x = -1$ 24

$x^2 = 1$




25

$$\frac{\frac{1}{2}\pi\left(\frac{d}{2}\right)^2}{\frac{1}{2}\pi\left(\frac{d}{2}\right)^2 + hd} = \frac{1}{3}$$

$$\frac{\frac{1}{2}\pi\left(\frac{d}{2}\right)^2 + hd}{\frac{1}{2}\pi\left(\frac{d}{2}\right)^2} = 3$$

$+$

$$1 + \frac{h\cancel{d}}{\frac{\pi}{8} d^{\cancel{2}}} = 3$$

$$\frac{8h}{\pi d} = 2$$

$$\frac{h}{d} = \frac{\cancel{2}\pi}{\cancel{8}\,4} = \frac{\pi}{4}$$

26.

$$\begin{bmatrix} 1 & -1 \\ 1 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = 2 \begin{bmatrix} x \\ y \end{bmatrix}$$

$$x - y = 2x$$

$$x + 3y = 2y$$

$$x + y = 0$$

$$x + y = 0$$

27 operations commutative $xy = yx$

associative $(xy)z = x(yz)$

abb
bba
bab
bbb
aaa
aab
aba
baa

$(a \times b) \times b$

$a \times b = a$

$a \times (b \times b)$

$a \times a = b$

29

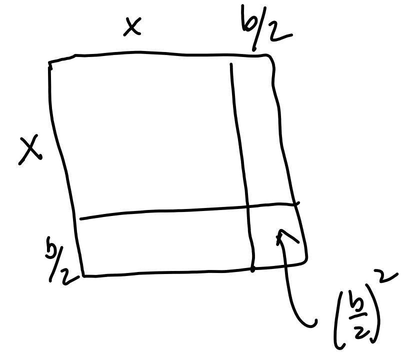
$$x^2 + 2x + y^2 = 0$$

$$\underbrace{x^2 + 2x + 1^2} - 1^2 + y^2 = 0$$

$$(x+1)^2 - 1 + y^2 = 0$$

$$(x+1)^2 + y^2 = 1$$

$(-1, 0)$ $r = 1$




32

$y = x^3 + 1$

$x = y^3 + 1$ $\Rightarrow$ $x - 1 = y^3$

$\sqrt[3]{x-1} = y$

33

$$\begin{bmatrix} 3 & -3 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \end{bmatrix}$$

$$I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\underset{A}{\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}}\underset{B}{\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}}$$

$$\star\ 1+2+3+4+\dots+n = \frac{n(n+1)}{2}$$

$$1^2+2^2+3^2+4^2+\dots+n^2 = \frac{n(2n+1)(n-1)}{2}$$ ← check this

$$5\left(\frac{1}{6}\right)+\frac{2}{6} = \frac{7}{6} \neq 1$$

$$5p+2p = 7p = 1$$

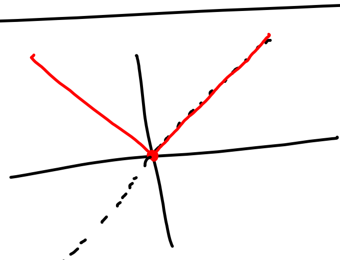
$$p = \frac{1}{7}$$

$1,3 \rightarrow \frac{2}{7}\cdot\frac{1}{7}$ ) +

$3,1 \rightarrow \frac{1}{7}\cdot\frac{2}{7}$ ) +

$2,2 \rightarrow \frac{1}{7}\cdot\frac{1}{7}$

---



$f\left(\frac{r}{s}\right) = \frac{r}{s}$ $\quad f(x) = x \quad f(t) = t$

$f(|x|) = |x|$

1) a)

| year | pop. |
|---|---|
| 2000 | 7 |
| 2001 | 14 |
| 2002 | 28 |
| 2003 | 56 |
| 2004 | 112 |
| 2005 | 224 |

b) i) 7, 14, 28, 56, 112, ...

ii) $\left.\begin{aligned} a_0 &= 7 \\ a_n &= a_{n-1} \cdot 2 \end{aligned}\right\}$

iii) $a_n = 7 \cdot 2^n$

iv) $a_n > 1000$

$7 \cdot 2^n > 1000$

$2^n > \frac{1000}{7}$

$2^n > 142.8$

$n > 7.16$

IT WOULD TAKE 8 (OR MORE PRECISELY 7.16 years).

c)

$$c + cr + cr^2 + \dots + c r^{\overset{20}{n-1}} = \frac{c(1 - r^{\overset{21}{n}})}{1 - r}$$

$\underset{7}{c}\ \underset{2}{r}$

(i) $= \frac{7(1 - 2^{21})}{1 - 2} \cong$ 14 million

(ii) $a_{21} + a_{22} + \dots + a_{40}$

$$\underbrace{a_0 + a_1 + \dots + a_{20}}_{\text{part (i)}} + a_{21} + \dots + a_{40} = \frac{7(1 - 2^{41})}{1 - 2}$$

$$\frac{7(1-2^{41})}{-1} - \frac{7(1-2^{21})}{-1} = 7\left[(2^{41}-1) - (2^{21}-1)\right]$$

$$= 7\left[2^{41} - 2^{21}\right]$$

2

(a) $3t + t = 1$

$4t = 1$

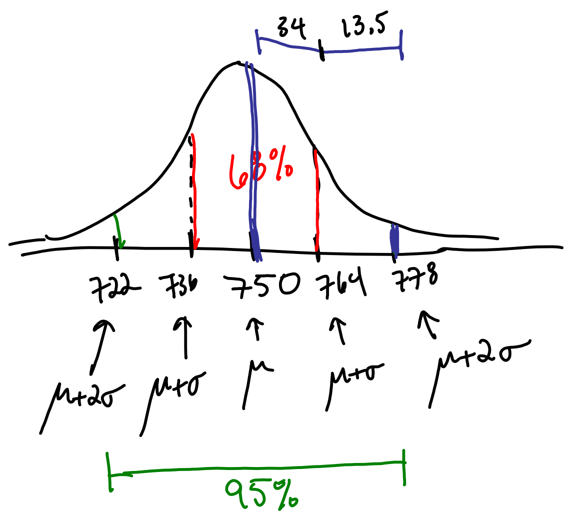
$t = \frac{1}{4}$ $\Rightarrow$ PROBABILITY OF HEADS IS $\frac{3}{4} = p$

(b)

$$\mu = n \cdot p = 1000 \cdot \frac{3}{4} = 750$$

$$\sigma = \sqrt{n \cdot p \cdot (1-p)} = \sqrt{1000 \cdot \frac{3}{4} \cdot \frac{1}{4}} \cong 14$$

(c)




(d) 32% LIES OUTSIDE THE INTERVAL [736, 764], AND HALF IS BELOW 736 : $\frac{32\%}{2} = 16\%$

(e) $E = 100 \cdot (.475) + (-90)(.525) = 0.25$

$$\left( 500 \cdot 0.25 = \$125 \right)$$

---

③ $\dfrac{1}{13}$

$$
\begin{array}{r}
.07692307 \\
13 \overline{)\,1.00000000} \\
\underline{91} \\
90 \\
\underline{78} \\
120 \\
\underline{117} \\
30 \\
\underline{26} \\
40 \\
\underline{39} \\
100 \\
\underline{91} \\
9\ldots
\end{array}
$$

FOR ANY NATURAL NUMBER $n$, THERE ARE $n$ NATURAL NUMBERS $\leq n$, AND SO ONLY THIS MANY POSSIBILITIES FOR THE ELEMENTS IN THE SEQUENCE OF REMAINDERS OBTAINED IN LONG DIVISION.

IF THE REMAINDER AT ANY STAGE IS ZERO, THE SEQUENCE TERMINATES, AS DOES THE DECIMAL EXPANSION.

IF ZERO DOES NOT OCCUR, THE SEQUENCE MUST REPEAT IN $n-1$ TERMS SINCE THERE ARE ONLY $n-1$ NUMBERS IN $\{1, 2, \ldots, n-1\}$, AND ONCE A NUMBER REPEATS, THE WHOLE SEQUENCE MUST BECAUSE THE LONG DIVISION PATTERN WILL REPEAT.

(4)

(a)

| $\oplus$ | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| 0 | 0 | 1 | 2 | 3 | 4 | 5 |
| 1 | 1 | 2 | 3 | 4 | 5 | 0 |
| 2 | 2 | 3 | 4 | 5 | 0 | 1 |
| 3 | 3 | 4 | 5 | 0 | 1 | 2 |
| 4 | 4 | 5 | 0 | 1 | 2 | 3 |
| 5 | 5 | 0 | 1 | 2 | 3 | 4 |

(b) YES, COMMUTATIVE SINCE DIAGRAM IS SYMMETRIC

$$(a \oplus b) \oplus c \stackrel{?}{=} a \oplus (b \oplus c)$$

$$(a+b \bmod 6) + c \bmod 6 \stackrel{?}{=} a + (b+c \bmod 6) \bmod 6$$

yes.

(c) $0$ IS $\oplus$-IDENTITY.

FOR $a$, $a^{-1}$ IS THE NUMBER SO THAT

$$a \oplus a^{-1} = 0$$

$0^{-1}$ IS $0$
$1^{-1}$ IS $5$
$2^{-1}$ IS $4$
$3^{-1}$ IS $3$
$4^{-1}$ IS $2$
$5^{-1}$ IS $1$