DC FELLOWS PROGRAM
MATH IMMERSION SUMMER COURSE

MATRIX ALGEBRA
JUNE 10, 2008

1. Multiple choice problems

(1) Given $\triangle ABC$ with vertices $A(0,0)$, $B(3,0)$, and $C(0,4)$ in the xy-plane, which of the following matrix transformations represents a dilation of $\triangle ABC$ with center $(0,0)$ and scale factor 3?

(a) $\begin{pmatrix} 0 & 3 \\ 0 & 3 \end{pmatrix} \begin{pmatrix} 0 & 3 & 0 \\ 0 & 0 & 4 \end{pmatrix}$

(b) $\begin{pmatrix} 3 & 3 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} 0 & 3 & 0 \\ 0 & 0 & 4 \end{pmatrix}$

(c) $\begin{pmatrix} 3 & 3 \\ 3 & 3 \end{pmatrix} \begin{pmatrix} 0 & 3 & 0 \\ 0 & 0 & 4 \end{pmatrix}$

(d) $\begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix} \begin{pmatrix} 0 & 3 & 0 \\ 0 & 0 & 4 \end{pmatrix}$

$$\begin{pmatrix} 5 & 1.5 \\ 2 & d \end{pmatrix}$$

(2) For what value of d is the 2×2 matrix not invertible?

(a) -0.6
(b) 0
(c) 0.6
(d) 3

(3) Solve the system for y:

$$\begin{aligned} 7x - 8y + 5z &= 5 \\ -4x + 5y - 3z &= -3 \\ x - y + z &= 0 \end{aligned}$$

(a) -2
(b) -1
(c) 0
(d) 1

(4) The orthogonal projection of 3-space onto the xy-plane takes the point (x, y, z) onto the point $(x, y, 0)$. This transformation can be represented by the matrix equation

$$M \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} x \\ y \\ 0 \end{pmatrix},$$

where M is which of the following matrices?

(a) $\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

(b) $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$

(c) $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

(d) $\begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

(5) For what value of x is the matrix NOT invertible? $\begin{pmatrix} 1 & 4 \\ x & 6 \end{pmatrix}$

(a) -1.5
(b) 0
(c) 1.5
(d) 2

(6) The scalar multiple of the number 3 with the matrix $\begin{pmatrix} 2 & 1 \\ 3 & 5 \end{pmatrix}$

(a) 33

(b) $\begin{pmatrix} 6 & 1 \\ 9 & 5 \end{pmatrix}$

(c) $\begin{pmatrix} 2 & 3 \\ 3 & 15 \end{pmatrix}$

(d) $\begin{pmatrix} 6 & 3 \\ 9 & 15 \end{pmatrix}$

(7) The product of two matrices can be found only if

(a) The number of rows in the first matrix is equal to the number of rows in the second matrix.
(b) The number of columns in the first matrix is equal to the number of columns in the second matrix.
(c) The number of columns in the first matrix is equal to the number of rows in the second matrix.
(d) The number of rows in the first matrix is equal to the number of columns in the second matrix.

(8) a problem

(a) choice1
(b) choice2
(c) choice3
(d) choice4

(9) a problem

(a) choice1
(b) choice2
(c) choice3
(d) choice4

(10) a problem

(a) choice1
(b) choice2
(c) choice3
(d) choice4

(11) a problem

(a) choice1
(b) choice2
(c) choice3
(d) choice4

(12) a problem

(a) choice1
(b) choice2
(c) choice3
(d) choice4

(13) a problem

(a) choice1
(b) choice2
(c) choice3
(d) choice4

(14) a problem

(a) choice1
(b) choice2
(c) choice3
(d) choice4

(15) a problem

(a) choice1
(b) choice2
(c) choice3
(d) choice4

2. Structured response problems

(1) Let $A = \begin{pmatrix} 2 & 1 \\ 3 & 5 \end{pmatrix}$, and $B = \begin{pmatrix} 5 & 0 \\ 1 & 2 \end{pmatrix}$. Find $A + B$, $B - 3A$, $A \cdot B$, and $B \cdot A$.

Does $A \cdot B = B \cdot A$? What does this tell you about matrix multiplication?

(2) Find the matrix X:

$$3X + \begin{pmatrix} 1 & 5 & 7 \\ 4 & 1/2 & 0 \end{pmatrix} = \begin{pmatrix} 2 & -8 & 1/3 \\ 4 & 2.5 & 3 \end{pmatrix}$$

(3) Let $A = \begin{pmatrix} 2 & 1 \\ 4 & -3 \\ 0 & 5 \end{pmatrix}$, $B = \begin{pmatrix} -2 \\ 3 \end{pmatrix}$, and $C = \begin{pmatrix} 1 & 2 & 3 \end{pmatrix}$. Find all existing products.

(4) Consider the following matrix equation.

$$\begin{pmatrix} -2 & 1 & 0 & 1 \\ 4 & 3 & -2 & 1 \\ 0 & 0 & 5 & -6 \\ 3 & 2 & -1 & 0 \end{pmatrix} \cdot X = \begin{pmatrix} -4 \\ -2 \\ 0 \\ 1 \end{pmatrix}$$

- What are the dimension of X?
- Use your calculator to solve the matrix equation below for X. Explain your input, output, and the functions used.