DC FELLOWS PROGRAM
MATH IMMERSION SUMMER COURSE

PROOF PRACTICE
JUNE 10, 2008

(1) If x and y are positive real numbers and $x \neq y$, then

$$\frac{x}{y} + \frac{y}{x} > 2.$$

(2) (a) Assume a, b, and c are real numbers and state the converse of "If $a + b = c$ then $(a + b)^2 = c^2$."

(b) Show that the converse is not true by finding a counterexample.

(3) The Intermediate Value Theorem says that if a real-valued function f is continuous on the closed interval $[a, b]$ and N is a number strictly between $f(a)$ and $f(b)$, then there is a number c in (a, b) such that $f(c) = N$.

Use the Intermediate Value Theorem to show that $f(x) = x^3 + x - 1$ has a real root.

(4) Prove or disprove: If a, b, and c are integers such that a divides b, then a divides bc.