DC FELLOWS PROGRAM
MATH IMMERSION SUMMER COURSE

STRUCTURED RESPONSE PRACTICE
JUNE 6, 2008

(1) (PROBLEM) A gnat is randomly flying inside a spherical balloon of radius 8 inches. What is the probability that at any given time the gnat is a distance of more than 4 inches from the center of the balloon? Show how you determined the probability.

(2) (MODEL) A calculator company produces a scientific calculator and a graphing calculator. Long-term projections indicate an expected demand of at least 100 scientific and 80 graphing calculators each day. Because of limitations on production capacity, no more than 200 scientific and 170 graphing calculators can be made daily. To satisfy a shipping contract, a total of at least 200 calculators must be shipped each day.

If each scientific calculator sold results in a $2 loss, but each graphing calculator produces a $5 profit, how many of each type should be made daily to maximize net profits?

(3) (PROOF) A function g defined for all real numbers is called an *even* function if $g(-x) = g(x)$ for all real x.

A function h defined for all real numbers is called an *odd* function if $h(-x) = -h(x)$.

(a) If f is any function defined for all real numbers, prove that $f(x)+f(-x)$ is an even function.
(b) If f is any function defined for all real numbers, prove that $f(x)-f(-x)$ is an odd function.
(c) Prove that any function f defined for all real numbers can be written as the sum of an even function and an odd function.

(4) (PROBLEM) For each trip of $\frac{1}{2}$ mile or less, a certain taxi service charges \$1.00; for each trip of more than $\frac{1}{2}$, the taxi service charges \$1.00 for the first $\frac{1}{2}$ mile and \$0.50 for each additional mile or fraction of a mile.

(a) (i) What amount does the taxi service charge for a trip of 2 miles? Show your work.

(ii) What amount does the taxi service charge for a trip of $\frac{3}{5}$ mile? Show your work.

(b) For every number x, let $\lceil x \rceil$ denote the least integer greater than or equal to x. Using this notation, express the amount, y, in dollars that the taxi service charges for a trip of x miles as a function of x. Show your work.

(c) In the rectangular coordinate plane, draw the graph of the function you found in part 4b.