# Quantum Teleportation

By Alexa Becker and Hui Jiang

# What is teleportation?

# History

- First paper published on the subject was in 1993
- Published by C. H. Bennett, G. Brassard, C. Crépeau, R. Jozsa, A. Peres and W. K. Wootters in 1993
- First implementation was with single photons, but has since been demonstrated with atoms, ions, electrons, and superconducting circuits
- Proven to teleport one or more qubits of information between two entangled atoms, but has not been achieved with anything larger

# Background

- Process by which quantum information can be transmitted from one location to another, with the help of classical communication and previously shared quantum entanglement between the sending and receiving location
- Quantum teleportation is not a form of transportation, but of communication; it provides a way of transporting a qubit from one location to another, without having to move a physical particle along with it
- Teleportation protocol requires that an entangled quantum state or Bell state be created, and its two parts shared between two locations

# Problems With Quantum Teleportation

1. **No-cloning theorem**
2. **No-deleting theorem**
3. **EPR Paradox**

# No-Cloning Theorem

**It is impossible to create an identical copy of an arbitrary unknown quantum state.**

- The state of one system can be entangled with the state of another system.
- For instance, one can use the controlled NOT gate and the Hadamard gate to entangle two qubits, but this is not cloning.
- Cloning is a process whose result is a separable state with identical factors

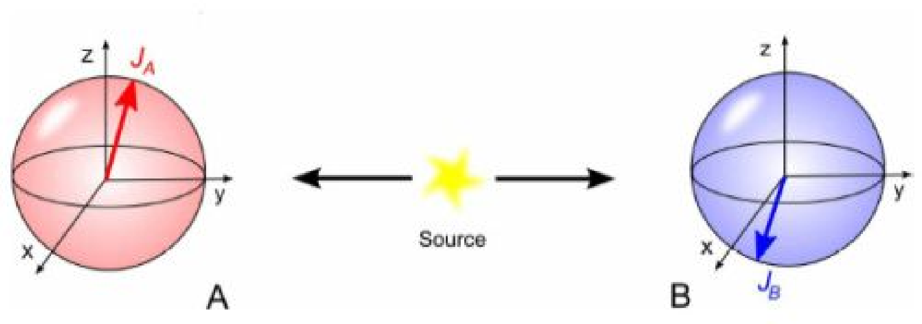
# No-Deleting Theorem

**Given two copies of some arbitrary quantum state, it is impossible to delete one of the copies.**

- It is a time-reversed dual to the no-cloning theorem
- If it were possible to delete an unknown quantum state, then, using two pairs of EPR states, we could send signals faster than light.
- The process of quantum deleting takes two copies of an arbitrary, unknown quantum state at the input port and outputs a blank state along with the original.

$$U|\psi\rangle_A|\psi\rangle_B|A\rangle_C = |\psi\rangle_A|0\rangle_B|A'\rangle_C$$

# EPR Paradox



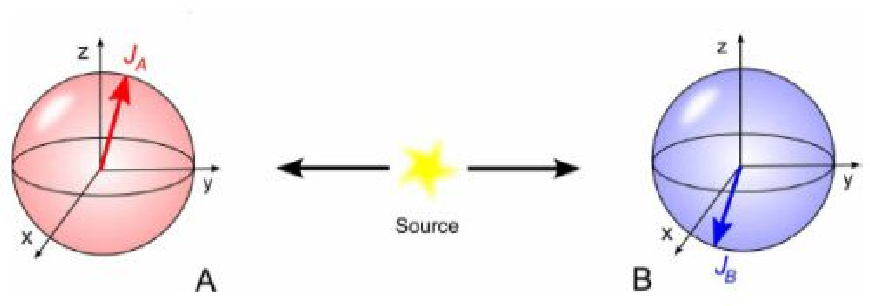
Created in 1935 by Albert Einstein, Boris Podolsky, and Nathan Rosen, hence "EPR".

**Wave function**: describes the quantum state of an isolated system of one or more particles. There is *one* wave function containing all the information about the entire system, not a separate wave function for each particle in the system.

**Heisenberg's uncertainty principle**: the more precisely the position of some particle is determined, the less precisely its momentum can be known.

Demonstrates that the **wave function** does not provide a complete description of physical reality because particles can interact in such a way that it is possible to measure both their position and momentum more accurately than **Heisenberg's uncertainty principle** allows.

# EPR Paradox



It is one thing to say that physical measurement of the first particle's momentum affects uncertainty in its *own* position, but to say that measuring the first particle's momentum affects the uncertainty in the position of the *other* is another thing altogether.

Einstein, Podolsky and Rosen asked how can the second particle "know" to have precisely defined momentum but uncertain position?

-Since this implies that one particle is communicating with the other instantaneously across space, faster than light, this is the "paradox".

# Example

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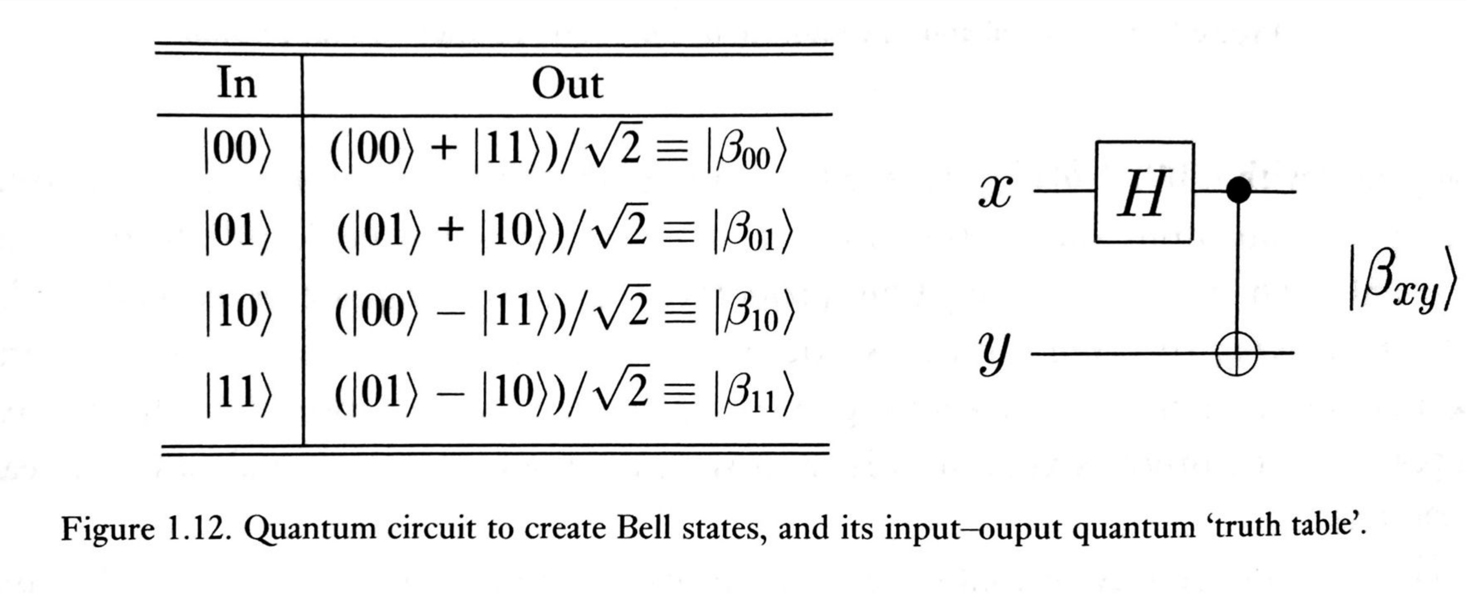
Bell states:

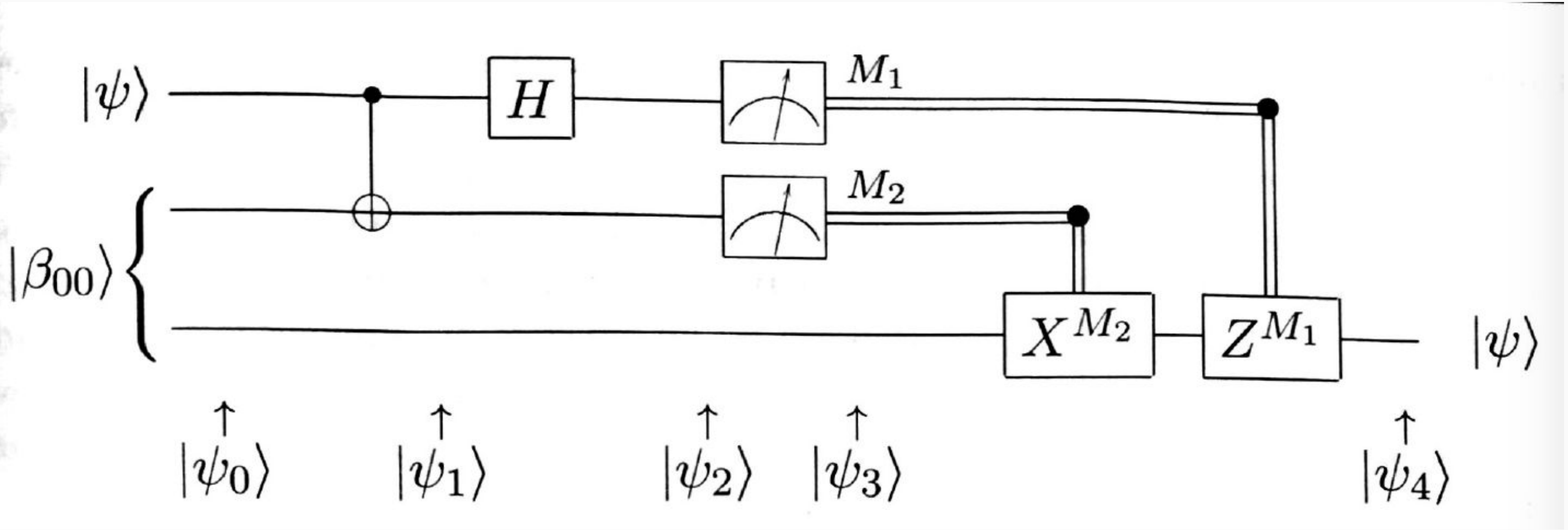


| In | Out |
|---|---|
| $\vert 00\rangle$ | $(\vert 00\rangle + \vert 11\rangle)/\sqrt{2} \equiv \vert\beta_{00}\rangle$ |
| $\vert 01\rangle$ | $(\vert 01\rangle + \vert 10\rangle)/\sqrt{2} \equiv \vert\beta_{01}\rangle$ |
| $\vert 10\rangle$ | $(\vert 00\rangle - \vert 11\rangle)/\sqrt{2} \equiv \vert\beta_{10}\rangle$ |
| $\vert 11\rangle$ | $(\vert 01\rangle - \vert 10\rangle)/\sqrt{2} \equiv \vert\beta_{11}\rangle$ |

Figure 1.12. Quantum circuit to create Bell states, and its input–ouput quantum ‘truth table’.

# Examples

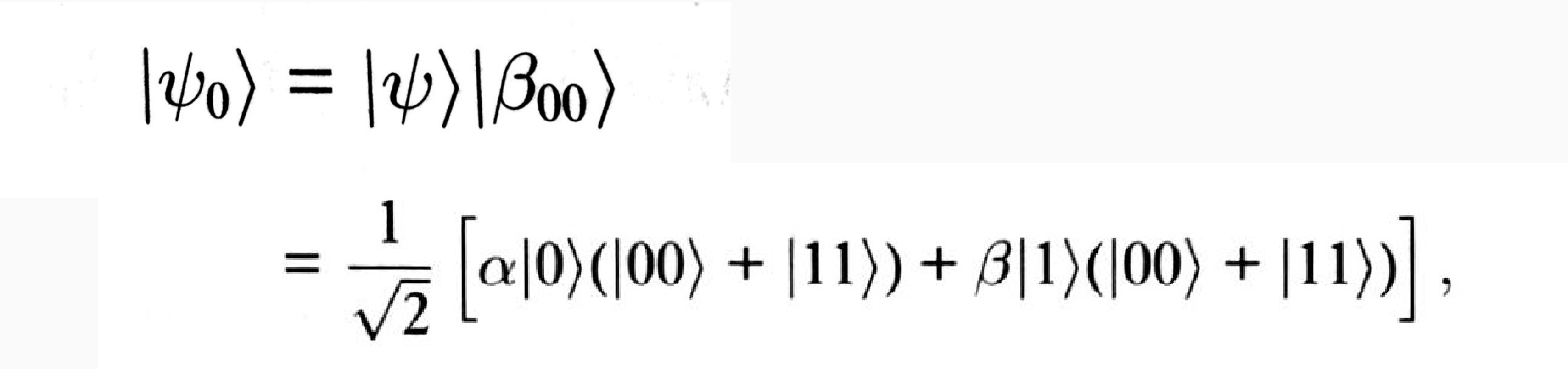
Alice uses quantum teleportation to send information to Bob.



The top two lines represent Alice's system, while the bottom line is Bob's system.

# Example

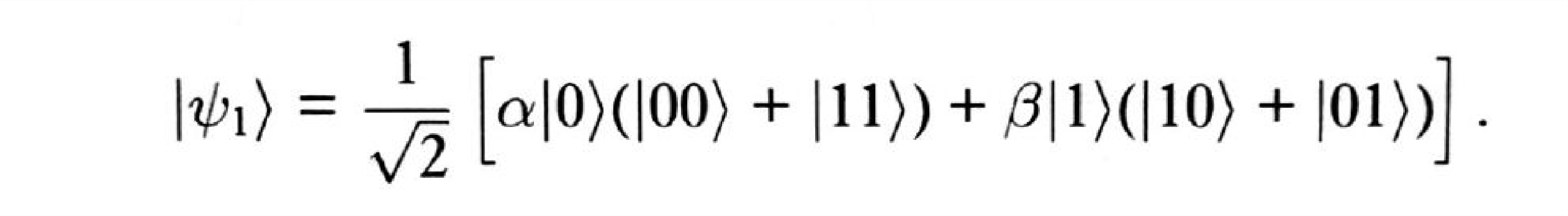
Input state:



$$
\begin{aligned}
|\psi_0\rangle &= |\psi\rangle|\beta_{00}\rangle \\
&= \frac{1}{\sqrt{2}}\left[\alpha|0\rangle(|00\rangle + |11\rangle) + \beta|1\rangle(|00\rangle + |11\rangle)\right],
\end{aligned}
$$

# Example

Alice sends her qubit through a CNOT gate, obtaining



$$|\psi_1\rangle = \frac{1}{\sqrt{2}}\left[\alpha|0\rangle(|00\rangle + |11\rangle) + \beta|1\rangle(|10\rangle + |01\rangle)\right].$$

# Example

Then sends the first qubit through a Hadamard gate, obtaining

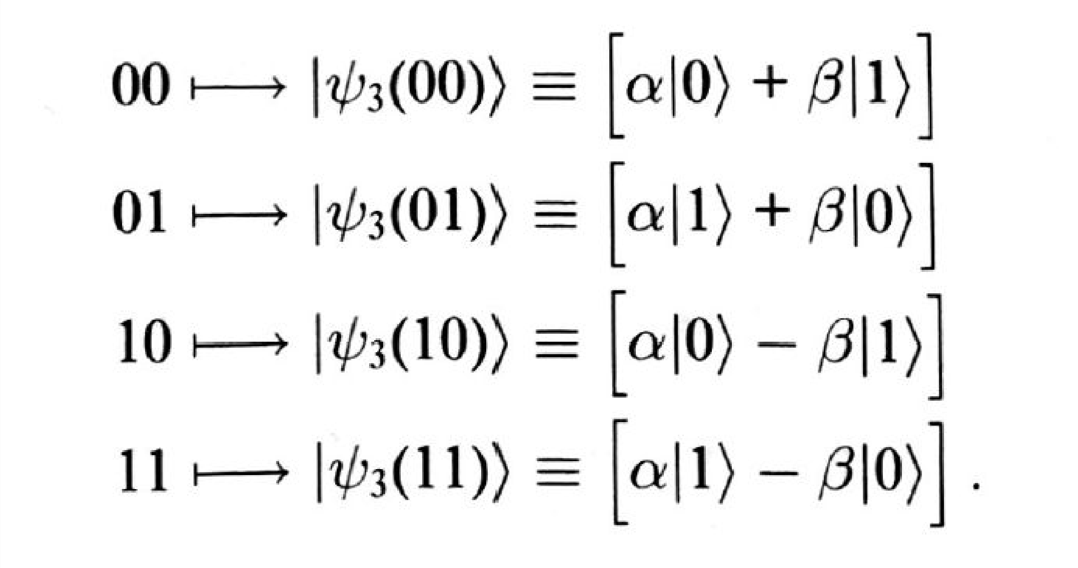
$$|\psi_2\rangle = \frac{1}{2}\Big[\alpha(|0\rangle + |1\rangle)(|00\rangle + |11\rangle) + \beta(|0\rangle - |1\rangle)(|10\rangle + |01\rangle)\Big] .$$

Re-write in the following way, obtaining

$$\begin{aligned} |\psi_2\rangle = \frac{1}{2}\Big[&|00\rangle\left(\alpha|0\rangle + \beta|1\rangle\right) + |01\rangle\left(\alpha|1\rangle + \beta|0\rangle\right) \\ &+ |10\rangle\left(\alpha|0\rangle - \beta|1\rangle\right) + |11\rangle\left(\alpha|1\rangle - \beta|0\rangle\right)\Big] . \end{aligned}$$

# Example

The result of Alice's measurement:



$$\begin{aligned}
00 &\longmapsto |\psi_3(00)\rangle \equiv \Big[\alpha|0\rangle + \beta|1\rangle\Big] \\
01 &\longmapsto |\psi_3(01)\rangle \equiv \Big[\alpha|1\rangle + \beta|0\rangle\Big] \\
10 &\longmapsto |\psi_3(10)\rangle \equiv \Big[\alpha|0\rangle - \beta|1\rangle\Big] \\
11 &\longmapsto |\psi_3(11)\rangle \equiv \Big[\alpha|1\rangle - \beta|0\rangle\Big] .
\end{aligned}$$

# Records

First experiment in 1998 verified the initial predictions, and the distance of teleportation was increased in August 2004 to 600 meters, using optical fiber.

Since then, the record distance of teleported photonic qubits has been gradually increased to 16 km, then to 97 km, and is now 143 km (89 mi), set in open air experiments done between two of the Canary Islands.

In September of 2015, Physicists at the National Institute of Standards and Technology set a new record using a supercooled nanowire that reached the distance of 101 km or 63 miles (4 times the previous record), the ability to do so over conventional fiber-optic lines compared to free space offers more flexibility for network design.