

The Fusion Project: Bridging the Art Museum and the Middle School Math Classroom

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The Fusion Project is a distinct program of the University of San Francisco's College of Arts and Sciences with the support of the School of Education. It was created by Philip Wagner and is directed at USF by Benjamin Wells. With the support of the Fine Arts Museums of San Francisco, its goal is to bring art to the math classroom and math students to the art museum, enhancing existing curricula in order to improve basic and advanced skills, standard-oriented test scores, and students' interest in mathematics.

Keywords: mathematics education, math in art museums; art in math classrooms; museum-school mathematics collaboration

AMS Subject Classification: 97D40, 97B50, 97U99

1. History and background

In the new century, the United States government recognized the need to improve knowledge and skills in language and mathematics, and so the US authorized the No Child Left Behind (NCLB) program in 2002. Under this program the performance of public schools has been judged on standardized test scores in language and mathematics. Review of the well-publicized testing results over the years indicates that that California students rank below their peers in other states and US students rank below their fellows in other countries based on similar testing. San Francisco schools in particular have missed the bar set by NCLB every year.¹ San Francisco and California and the USA are not unique or singular; they are just where we are and where we begin. Later we shall discuss extensions to other settings, geographic and pedagogic, but from the outset, many readers will recognize that this is not a problem peculiar to a small region.

Philip Wagner conceived The Fusion Project [13] (FP) as a counterbalance to the embarrassingly poor performance of United States middle school students² in mathematics. The main idea of FP is to engage middle school students in learning particular critical math concepts by discovering them displayed in artwork and architectural components of a major art museum. As a retired American businessman with extensive international experience, Wagner could not believe that his own nation's students were put at such a disadvantage for global (and local) citizenship. Well educated by the standards of his generation, he knew that the skills necessary

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¹ According to a September 2008 San Francisco Unified School District press release [17], "SFUSD remains under NCLB program improvement status." The NCLB measure of overall Adequate Yearly Progress shows California going downhill from 67% of schools in 2006–07 to 52% in 2007–08 [16].

² For example, the recent results [3] show only 30% of California middle schools are performing at or above the state's own standards. But to give some perspective, that is a threefold improvement in the last decade, and high schools are less than 18% passing.

to achieve success in a global economy had changed. The challenges faced by students today are greater, at the same time their preparation for them is weaker.

The serious problem of poor math comprehension in American classrooms¹ suggests early intervention, well before high school, where many students fail exit exams required for diplomas. The California High School Exit Exam in mathematics, for example, is based on middle-school standards, yet among first-time takers (in the 10th grade) of the Class of 2008, 25% failed.² Besides the demonstrable and well-documented low scores on statewide standardized tests, there is a general torpor that sets in among middle school math students. This is widely reported by teachers of 7th grade through first-year college classes, and it should not be dismissed as anecdotal. How can we enhance and maintain interest among students who do not recognize the beauty and importance of math in their lives and their futures in the world economy? Wagner had the inspiration that art could anchor the interest of math students, especially those for whom the standard text applications are distant and uninvolved. And where better to find art than in art museums? His first collaborative exploration led to an ongoing connection between what he had now named The Fusion Project and the Fine Arts Museums of San Francisco (FAMSF), whose de Young Museum had recently reopened in a spectacular new building in Golden Gate Park.

Thinking that the author of *Art and Physics* [21] would have an affinity for FP, Wagner contacted Leonard Shlain, a San Francisco Bay Area surgeon who was avowedly and provably nonexpert in both of the title fields. But Shlain was known for the type of bridge building that FP seeks. Although he was enthusiastic about the concept, he was involved in a new book project, *Leonardo's Brain*,³ and he referred Wagner to Benjamin Wells at the University of San Francisco (USF). Wells had hosted the author for several talks in his courses melding math, science, religion, and art. Thus he too had a proclivity for the crossover at the heart of FP. In addition, he had trained junior high school students to teach entire math classes for eight years through programs at the University of California, Berkeley.

1.1. The FP context

FP seeks to demonstrate that some children can improve their interest and understanding in math at a critical stage. More importantly, we hope to show that we can improve scores for some students on certain math concepts that are always included on a state's standardized tests and which a significant number of students do not answer correctly. Please understand from the outset that no school programs have been implemented yet, so there is no question of presenting classroom results by data or anecdote.

Our interviews with school principals, educators, parents, and students confirmed that our public schools face difficulty in meeting the challenge of improved mathematics learning with an accompanying failure to stimulate the imagination of middle school students in mathematics.

Moreover, we have learned that an individual teacher's professional evaluations often hinge on improving student performance on these tests. It is not an understatement that most middle school math teachers teach out of fear of their classes' test results. And yet, for example, the California scores continue below acceptable levels⁴ in a state with the seventh largest GDP in the world in 2007.⁵

¹ According to [20], the "National Assessment of Educational Progress (NAEP), also known as 'the Nation's Report Card,' is the only nationally representative and continuing assessment of what American students know and can do in reading and math. NAEP results are based on a sample of the state's students." Grade 8 NAEP math proficiency in the USA for the latest reported year of 2007 is 31%.

² See CAHSEE results at [4].

³ In fall 2008, with the book well underway, Shlain was diagnosed with brain cancer. Notwithstanding the illness and the irony, Leonard was able to complete *Leonardo's Brain* before he passed away in spring 2009. See <http://leonardshlainsbrain.com> for more information.

⁴ See [5] for 2008, and see previous Standardized Testing and Reporting (STAR) summaries for earlier years.

⁵ According to the International Monetary Fund, World Bank, and CIA World Factbook (for countries' Gross Domestic Product) [30] and the United States Bureau of Economic Analysis [23], California's 2007 Gross State Product of 1.8×10^9

1.2. *Where is the data?*

Something is obviously wrong. How can it be set right? The obvious starting point would be to identify those concepts least understood by students, at least according to mandated statewide testing. We assumed this information would be readily available from school district or states. Amazingly, we were wrong. While the State of California releases the total scores of the math test (for example [5]), it does not identify those math concept areas which are most frequently difficult for students, nor does it summarize test results by individual topics, much less questions. Indeed, the information is treated as classified, even for the math teachers themselves. Before we discuss the research that led to the solution of this problem and the foundation of FP's proposal, there is the story of Wagner's taking FP international.

2. The BIRS workshop and its impact on FP

Wagner and Wells independently learned of the Banff International Research Station for Mathematical Innovation and Discovery (BIRS) workshop "Innovations in Mathematics Education via the Arts," to be held at the Banff Centre, Alberta, Canada in early 2007. Wagner understood it to be for mathematicians, educators, and artists, and he was none of these. Wells had other commitments at the time, but when he learned several of his colleagues in math and art would be there, he wrote the organizers and was immediately invited. So he encouraged Wagner to do the same, sending his own endorsement of FP to the organizers. To his dismay, the now-invited Wagner ended up on his own when Wells's schedule indeed precluded attendance. Confident as a businessman, he nevertheless saw little hope of making a competent presentation to the trivalent constituency of the workshop. Wells encouraged him and enriched his PowerPoint show with museum images. Wagner flew to Canada, honored by the invitation, but still worried.

2.1. *FP chosen for intense review*

In the eyes of the workshop participants, Wagner's earnest presentation and obvious concern for students' math performance¹ outweighed any hesitation at his being an outsider. To his surprise and delight, the concept of FP was identified as having significant potential and was chosen among many projects for focused review and development by a panel of his colleagues. The group comprised: (1) Nat Friedman, producer of many Art and Math conferences and founder of the International Society of the Arts, Mathematics, and Architecture (ISAMA); (2) Doris Schattschneider, math-art author (e.g., [18, 19]) and long-time trainer of middle school math teachers at Moravian College; and (3) Stewart Craven, a Canadian middle school math teacher and administrator.

The focus group endorsed the general plan of FP. They recognized the special environment of the Bay Area with its numerous art museums, but they also envisioned broader application of the ideas and materials FP could develop. They agreed that the project embodied many of the purposes and principles of the workshop. They cautioned against any notion of reform or even enrichment, as these can run counter to entrenched patterns in schools or suggest that FP is elitist. They warned that administrators can be the toughest hurdle. The consensus recommendation was to target middle school students, especially the 7th grade, since interest in math is normally gained or lost at this grade/age level.

would have put the state seventh in the list of countries by GDP, between Italy and Spain.

¹ There was objective evidence for the concern. At the time of his talk, the most recent STAR summary showed: "In math, 40 percent of students scored at grade level or beyond this year." [1]

2.2. *Ongoing connections with BIRS*

One of the strongest outcomes of the workshop and focus group has been the continued contact among participants concerning FP. Since that time, attendees have made substantial contributions to the framework and its development. We have consulted with Schattschneider long distance and in person and communicated frequently with Friedman and Craven.

3. *FP under development*

Buoyed by the positive reception at Banff and the multiple offers of further contact and support, we met with the FAMSF Education Department in early 2007. Through a grant by the Israel Family Foundation of Boulder, Colorado, FAMSF contracted with Wells to evaluate the suitability of their permanent art collections, particularly those of the de Young Museum, for FP implementation. He saw this primarily as a problem of aligning works of art with target standards in the California 7th grade mathematics curriculum. The collections would be usable if the result was a strong correspondence between the art and the standards.

3.1. *Alignments*

Previously Wells had had five years of experience performing standards alignments for the Center for Educational Policy Research at the University of Oregon. For example, their Standards for Success (S4S), a project of the Association of American Universities in partnership with The Pew Charitable Trusts, produced a list of Knowledge and Skills for University Success [8] endorsed by 28 leading research universities. Then S4S aligned the list with state high school standards and assessments [9]; Wells had participated in this phase. So the required analytical techniques and decision processes for making the desired associations between art works and mathematical outcomes were well practiced.

Seeing and classifying the mathematical content of a work of art is fun but challenging, and refinement improves with each revisit to the museum or the FAMSF online ImageBase [14]. But the greater challenge has been determining the types of mathematical skills most in need. As mentioned above, this is not a straightforward task. Indeed, at the same time schools and teachers are at the overall mercy of pooled test results, the actual student behavior on individual problems is jealously guarded—apparently neither teachers nor administrators are eager to have that kind of detail made known even if it is anonymous and aggregated.

3.2. *Teasing out the target standards*

Through serendipitous contacts involving a TV tutoring show, a teacher at a continuation school, and anonymous semiofficial core listings by test items, we were finally able to identify the concept areas optimal for achieving our objective of improving test scores. Wells's report [25] to FAMSF gives the results of analyzing two sets of standardized test data and aligning these with 7th grade math standards. The alignments then yielded *target standards*, identified by vertical low test scores on particular items. Of the 46 one-decimal-place entries (including some under the rubric of Mathematical Reasoning¹) in the California 7th grade content standards [6], we classified ten as targets, and another ten as nearly target. The following list summarizes these twenty, with the high targets in **bold**. Those marked with * are designated as key standards² for 7th grade in the Mathematics Framework for California Public Schools [7, p. 112].

¹ Because Mathematical Reasoning standards are overarching and not independent of content, there are no problems that address them directly. In a similar way, specific applications for them are not suggested in FP materials. They will be apparent from the questions asked and tasks suggested.

² It may be significant that only four strong targets are key standards, and another eleven key standards are not targets of either kind.

- (1) **NS1.2*** $+$, $\times$, $-$, $/$ rational numbers (integers, fractions, terminating decimals); $(a/b)^m$
- (2) **NS1.3** Convert fractions to decimals, percents; compute in estimations and applications
- (3) **NS1.6** Calculate the percentage of increases and decreases of a quantity
- (4) **NS2.2*** Add and subtract fractions by using factoring to find common denominators
- (5) **AF1.2** Use correct order of operations to evaluate algebraic expressions such as $3(2x+5)^2$
- (6) **AF1.3*** Simplify numerical expressions by applying laws of rational numbers, and justify
- (7) **AF1.5** Represent quantitative relations graphically; interpret the parts of a graph
- (8) **AF3.3*** Graph linear functions; note Δy is same for given Δx ; rise/run = slope
- (9) **AF3.4*** Plot quantities whose ratios are constant (ft/in); fit line, interpret slope as ratio
- (10) **AF4.1*** Solve 2-step linear equations and inequalities in one rational variable; interpret
- (11) **AF4.2*** Solve multistep problems involving rate-speed-distance-time and direct variation
- (12) **MG2.1** Use formulas to find perimeter and area of elementary 2- and 3-dim figures
- (13) **MG2.2** Estimate, compute area of complex, irregular 2-, 3-dim figures by decomposition
- (14) **MG2.3** Compute perimeter, surface area, volume of 3-dimensional objects obtained from rectangular solids; scale volumes and surface areas.
- (15) **MG3.1** Identify geometric figures; construct with compass and straightedge (altitude, midpoint, diagonal, bisectors, circle parts, etc.)
- (16) **MG3.2** Use coordinate graphs to plot simple figures; determine length, area; translated/reflected image
- (17) **MG3.6*** Identify elements of 3-dim objects; describe skew lines, 3-plane intersections
- (18) **MR1.1** Analyze problems from relationships, relevant/irrelevant and missing information, patterns
- (19) **MR2.3** Estimate unknown quantities graphically; solve for them by using logic, arithmetic, algebra
- (20) **MR2.4** Make and test conjectures by using both inductive and deductive reasoning

3.3. *Clustering into stories/encounters*

Works of art at the de Young Museum were surveyed for mathematical and pedagogical content. This linkage is explored in a later section. With the target standards at hand, their alignment with the mathematical classification of art works made a convincing argument that the de Young Museum was extremely well meant for collaboration with FP. In fact, the art afforded coverage of all target standards and other useful ones.

Now the question arose of how to take advantage of this.

By clustering the art works and math topics through additional alignments, we began to see a natural decomposition of both into seven groups, first called Seven Stories (because one initial effort resembled a visual storyboard treatment) and later renamed Seven Encounters, reflecting the instructional enhancements added subsequently:

- Encounter 1. Counting, adding, multiplying, grouping, distributing, and estimating
- Encounter 2. Fractions, percent, parts, and rhythm
- Encounter 3. Lines, slopes, intercepts, equations, and ramps
- Encounter 4. Edge lengths, surface area, and volume of various 3D objects
- Encounter 5. Lines and planes in space, ruled surfaces, and belts
- Encounter 6. Parallels, perspective, other projections
- Encounter 7. Uniformity, symmetry, chaos, confusion

The first three stories are more elementary and cover more basic standards. The last three are more exploratory and qualitative. The final result is a broad proposal for math content connecting the classroom and the de Young Museum. It can serve as a lesson-plan guide for development of modules related to the target standards and of explorations for more advanced students and classes. The reader will understand how the process of determining target standards, matching them to artwork, and forming clusters is easily adaptable to other collections of art. Later we'll

mention extensions to other media and venues. We now discuss how development has proceeded with the de Young Museum's collections.

3.4. *The Teachers Advisory Group*

With the help of FAMSF, invitations were mailed to a thousand teachers in the Bay Area. We selected five middle school math teachers in public, private, and parochial schools to join the FP Teachers Advisory Group (TAG). After initial FP training in late 2007 at the de Young Museum, the TAG communicated by email with us and with each other. Assignments of stories were made, and the teachers have submitted two to four sets of draft instructional materials currently being compiled and edited. Our suggested outline [26, pp. 16–17] for building out a story into an encounter was circulated as a plausible framework. In each of the stories, there is room for innovation on this structure, but these are the primary goals:

- *Interest and attractiveness.* Teachers should be able to imagine using the material and immediately see how they could work with it, and even better, improve it.
- *Clarity.* Teachers (and their students) must see concrete suggestions, even if they are about abstract concepts; there must be no handwaving, jargon, glibness, cuteness, smugness.
- *Consistency.* These materials are being prepared by a committee, and it is our determination not to edit away all individuality; after all, this is an appeal to individual teachers to catch hold for the ride; in the end, though, there has to be a steady rhythm in the presentation and the tasks. The variation in art, story, and math will add to the natural variety. A visible and useful structure can provide the consistency.
- *Focus.* Our primary aim is the improvement in mandated and identified standards. We want students to do better on standardized tests, but we want them to improve through interest, not drill. Beyond that, we automatically offer enrichment, variety, and innovation. Teachers looking for that will see it easily. We do not need to go into detail, but we need to indicate that this is not (or not merely) an opportunity for more drill.

For more information on the encounters with sample lesson plans, please see our papers [27, 28] for the Bridges Conference 2009.

4. Combining art and mathematics

4.1. *Art on the FP side*

Although FP is not intended to provide art instruction to teachers or support it directly in their classrooms, we need to recognize and enunciate the following principles.

The evaluation of art in the de Young Museum's permanent collections was based on its potential for teaching math. Most of the site-specific commissions are especially appropriate, and the entire world-class collection is a superlative math resource. Examples are given below.

The artistic values were not assessed, nor are they directly a part of FP methodology. No art teachers were invited to the Teachers Advisory Group for much the same reason. Our math classroom teachers are not required, expected, or recruited to teach art.

That said, the authors and our advisors love and appreciate art and are stimulated by the mathematics we see in art. This will be shared by those attracted to join FP. All consultants feel that the heart of the Fusion Project is the art. Some of us are collectors, some have used art in math and science courses, some have taught art, and some have been founders of the Art & Math movement. We are sensitive to the needs of museums and of art programs in schools as well, and we intend to provide structural suggestions for including an art education aspect in the Fusion Project.

In particular, although the workshops will not train the math teachers in art education, they will certainly introduce them to the artistic dimensions, through guided tours by museum docents and references to art interpretation.

Of course, the reason we are at the de Young Museum is because they see us as an important activity for their own art education effort. Similarly, we endorse the collaborative involvement of schools' art teachers in FP classrooms.

With no emphasis on the appreciation of the art, the works in the first stories become little more than mathematical tokens to be counted and measured (but see Encounters 5–7 for surface development, perspective, and procedural art, which are topics in art school courses). That is far from our goal. In fact, we believe that the interest in the mathematics that flows from the art will make the art that much more interesting and significant to the students, and we intend to help that flourish.

4.2. *Resources on the museum side*

To see how the art fits into the stories, the reader needs to know that there are five different aspects of the FAMSF art resources open to FP:

- Works of art in the permanent collections—the restriction aids stability of instructional materials, and access to the art is better;
- Cultural artifacts in the ethnological collections;
- Architectural elements (e.g., windows, staircases, furniture);
- Site-specific works and exhibits commissioned by FAMSF;
- Poster-sized prints of any image in ImageBase or 4D, the FAMSF internal high-resolution image database.

Each encounter involves all of these: the artwork under study may be monumental, may have cross-cultural features, may be part of the building or its furnishings, may visit the classroom as a poster. Based on FAMSF experience with their program Get Smart with Art @ the de Young [11], it is anticipated that several iterations of refinement will be required for a working set of encounter and enhancement materials. Once that result is produced, we can begin organizing workshops for teachers, evaluate the materials in classroom settings, obtain feedback from the users, and move to final printed products, with implementation already underway.

5. Four examples linking art and math at the de Young Museum

In order to clarify the use of the art in FP, we present four examples of works from the de Young Museum, discuss the math visible in the art, and suggest how a mathematical view can impact one's appreciation of the art. Although this discussion focuses on a single work from each of four encounters, there may be a dozen other pieces named by FP that broaden an encounter, and frequently one work enriches several.

5.1. **Strontium** (Gerhart Richter), *featured in Encounter 1*

This work begs counting the balls, yet thwarts it by its vast size and blurred images (see Fig. 1). The uniformity of the 130 panels invites an easy counting method by decomposition, but when the C-print panels are approached, the viewer can experience an unpleasant disorientation because of the defocused imagery. Among other styles, Richter is known for photorealism¹, but these *are* photographs. Yet this is not a boring repetition of identical boring images. It is instead

¹ For example, his *Lesende* at the San Francisco Museum of Modern Art.

a photorealistic representation of the crystal lattice of strontium titanate. The manipulated blurring points to the realism of probabilistic “boundaries” in quantum physics.

To Americans of a certain age, the title *Strontium* will recall the nuclear test fallout scare of the late '50s. Researchers around the world then detected elevated levels of the metal's radioactive isotope, strontium-90, which entered the global—and especially the American—food supply.

An artist unusually mindful of history, Richter might welcome that association. But his proposal for the mural mentions strontium titanate's unusually high index of refraction—it outshines diamonds in crystalline form—and its use in optics. He probably also likes the electron microphotograph as an image of a physical reality normally invisible. The scientific instrument's extension of unaided sensory awareness parallels that of the museum.[2]

The units that compose Richter's mural are photographs based in the realm of nanotechnology, which has tremendous resonance for San Francisco and the greater Bay Area, the capital of the high tech industry. The piece also relates wonderfully to the museum itself in that the pattern of circles throughout the mural is reminiscent of the perforated copper cladding on the new de Young building. —Daniell Cornell, Associate Curator of American Art, FAMSF, quoted in [10].

With regard to the mathematics implicit in the photographic array, the counting is obviously the most immediate but also the lowest level. On top of that are questions about translation and rotation invariance, checkerboard-type tasks, arithmetic and geometric problems associated with the latticework of Gaussian integers, and explorations of homage to the de Young's bossed copper skin apparent in this site-specific work. Perspective is also a welcome though unwitting partner here too (see Fig. 1).

Physical problems can arise from the crystalline aspect (including refraction).

There have been two other large works nearby that have offered much harder counting tasks, so *Strontium* acts as an touchstone for Encounter 1.¹

The math helps organize perceptions of the art and leads back to questions such as: why did the artist use this many panels and so many atoms? are the size and blur effective in drawing the viewer toward a stance that this is in fact art, not wallpaper?

5.2. Diagonal Freeway (Wayne Thiebaud), *featured in Encounter 3*

As depicted in Fig. 2, the road grades of 64% and 70% (slope = tangent of angle with frame bottom = 0.64 and 0.70) are impossible for highways (the steepest paved streets and roads seem to be less than 35%²). When the third dimension of depth is countenanced, then the freeway is seen to have single grade with a frightening perspective projection radiating from beyond the lower left corner. The slope is positive even if precipitous, but part of the disequilibrium of the viewer may be assigned to the steep descent from the right side, the cinematically ominous edge of the frame. Mostly it appears to be caused by the bland mechanistic background technocity with predictable two-point perspective applied. The eroded cliff both separates and joins the two artificial perspectives and suggests that nature trumps modern construction at the same time if fails to soften that improbable ramp, making it even more terrifying, or is it just absurd? Thiebaud offers this general view: “As far as I'm concerned, there is only one study and that is the way in which things relate to one another.”[22]

This painting supports calculating and comparing slopes in two dimensions and perspective constructions in three dimensions. FAMSF has several other of the artist's works that exploit steepness, some far more exaggerated than Fig. 2. Numerous works at the de Young Museum parallel these, and the architecture affords novel slopes, staircase ramps, and perspectives. *Diagonal Freeway* can serve as a linchpin in the study of both slope and perspective projection.

¹ Wells gave the counting problem to 5th graders, who submitted four different methods (plus one that only counted the perimeter).

² The former Guinness record of 38% for Baldwin Street, Dunedin, NZ, was based on an incorrect ratios $1:1.266 = 38^\circ$, a distortion of the also inaccurately quoted $1:2.66 = 38\%$; a municipal plaque on site indicates $1:2.86$ [29]

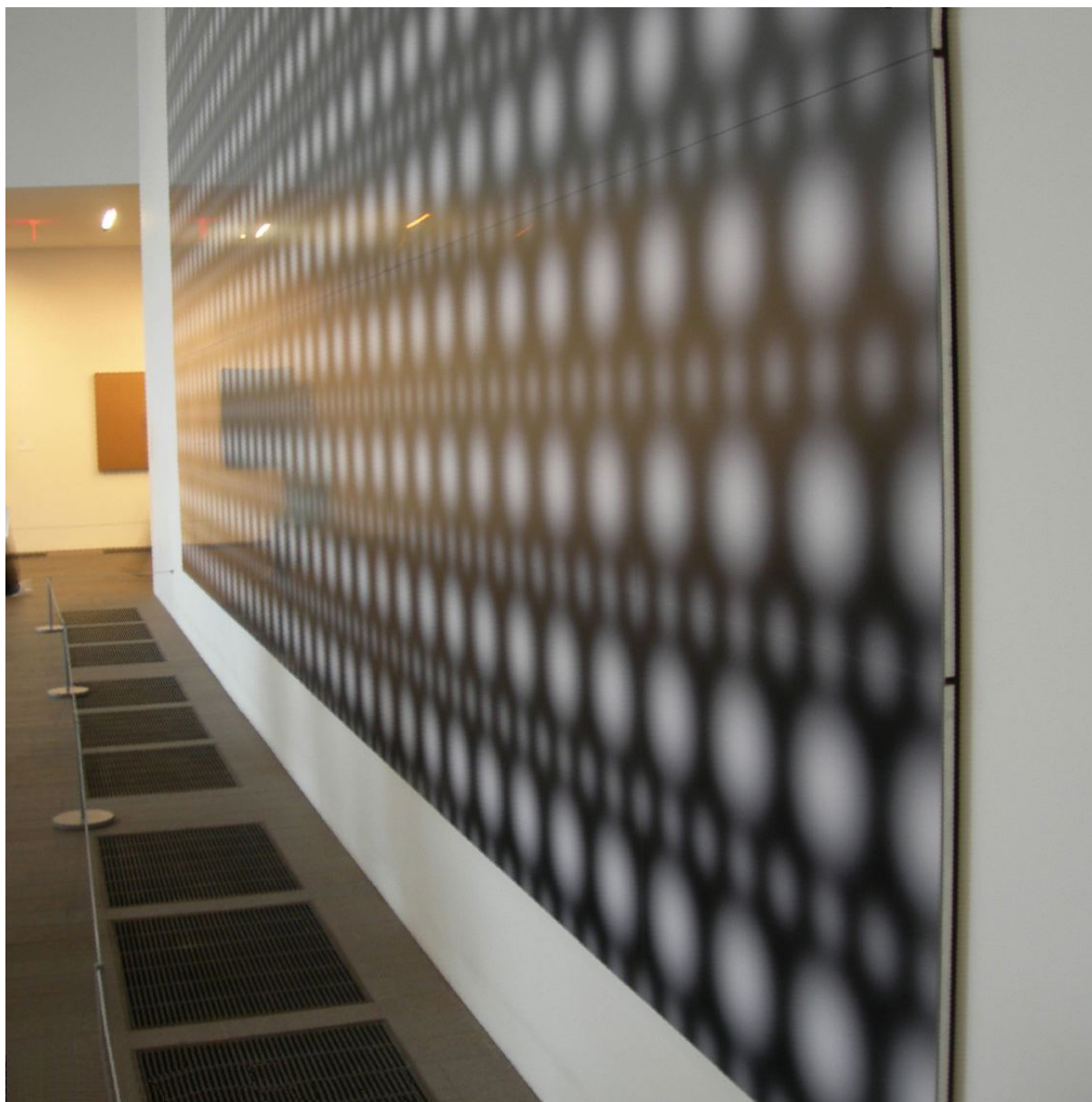


Figure 1. *Strontium* by Gerhard Richter. De Young Museum.

5.3. Three Gems (James Turrell), *featured in Encounter 6*

Because of its placement below grade in the garden, it is not possible to see this monumental installation in its entirety. It is to be explored or entered or joined, and not merely viewed. This instance of a Turrell skyspace³ offers an oculus at the top of a dome that encourages visitors to observe a small (or is it large?) disk of the sky. The hole is projected on the wall or tiled floor of the domed cylinder, sometimes as a bright sun, sometimes with clouds and fog (see Fig. 3). Natural light is combined with soft LED illumination at the base of the dome in a cycle of very slowly changing spectral colors (it speeds up at closing time). Light is a living artist here. Sound is also a contributor, with particularly attractive resonance for woodwinds⁴. The path to the dome passes through an underground tunnel to a red cylinder that forms a circular canyon around the domed structure. Access is not immediate, and most visitors enter in silence.

³ This is often charmingly misspelled as “skyscape.”

⁴ Wells listened to an impromptu bamboo flute, and there is a video clip of a recorder at [15].



Figure 2. *Diagonal Freeway* by Wayne Thiebaud. De Young Museum.

Reminiscent of a Pueblo kiva, the structure is also a representation of an internal, spiritual space at the same time it exposes the infinite sky as through a finite keyhole.

Musing on the motivations behind his skyspaces, Turrell said, “The sky always seems to be out there, away from us. I like to bring it down in close contact with us, so you feel you are in it. We feel we are at the bottom of this ocean of air; we are actually on a planet. We have spent billions to go to the moon—we go to this lesser satellite called the moon and say we are in space, but we are in space right now; we just don’t feel ourselves to be in space. Some forms of art and some forms of spirituality do give us that sense.” He also spoke of using light, something most people take for granted, as a material to make art, and emphasized that light can be seen as a thing, not merely as a phenomenon.

When discussing the ideal viewer for his skyspaces, Turrell took the opportunity to lament the miniscule amount of time viewers tend to spend with art these days, adding that he is not exempt from this accelerated culture. “I feel my work is made for one being, one individual,” he said. “You could say that’s me, but that’s not really true. It’s for an idealized viewer. Sometimes I’m kind of cranky coming to see something. I saw the Mona Lisa when it was in L.A., saw it for 13 seconds and had to move on. But, you know, there’s this slow-food movement right now. Maybe we could also have a slow-art movement, and take an hour.”[12]

The cylinders, disks, annuli, and hemisphere formed by walls, tiled floor, oculus, paving, and dome invite simple geometric problems that are nontrivial because most of these structures cannot be measured as directly as many of the other collection and building features. But thinking about such questions can also deepen the observer’s awareness of how this installation (or sculpture?) thwarts simple characterization. More predictable than a Serra torqued ellipsoidal cylinder, *Three Gems* orients the visitor, but plays a welcoming game with rules of nonstandard proportions. The central blue-green disk in the floor echoes the oculus, but in fact their sizes differ widely. The projection of the oculus can be also discussed with the concept of view window in computer graphics.

In addition, there are some nice combinatorial properties in the floor tiles, with fraction applications in the wall sections and pavers. More physical applications can relate to the acoustics

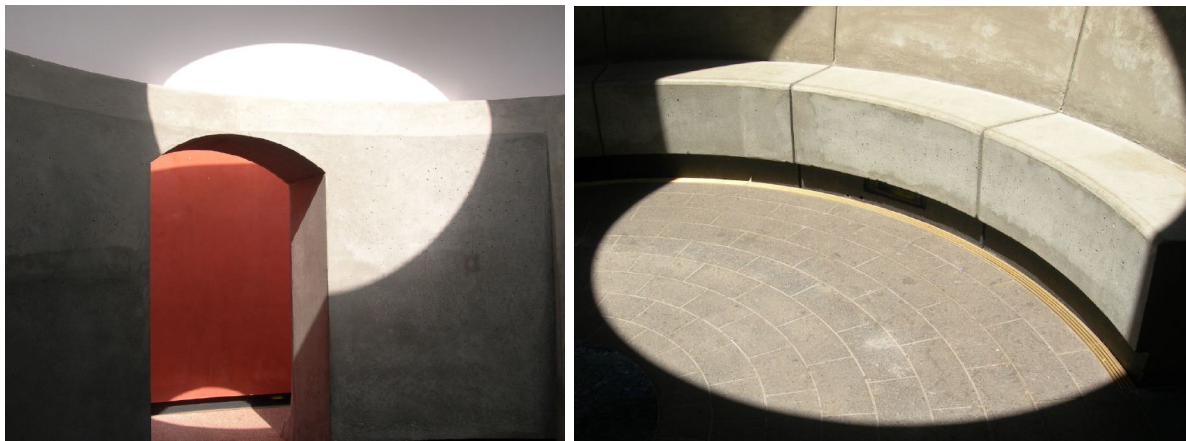


Figure 3. *Three Gems* by James Turrell. De Young Museum.

and the view out of this special observatory.

In this case, having an eye out for the math can help integrate one's experiences with this engaging and intimate, yet curiously aloof, piece of art.

5.4. Drawn Stone (Andy Goldsworthy), *featured in Encounter 7*

The entire entry courtyard is the canvas for this work, a designed crack through the sandstone pavers and all of the block benches (see Fig. 4). The crack appears both natural and artificial. In the view of the artist, it suggests an earthquake faultline (*Faultline* was his original name of the work). Experts in rock breaking say that angles around 120° are likely, but this crack steers a course horizontally or vertically for the most part. Wells opines that the edge cuts were made after the breaks, which can account for the control in the pavers, but the benches present more of a fabrication problem. Nothing suggests sawing, and Goldsworthy does not work that way; several accounts mention his breaking the pavers on site with a mallet. The homage to earthquakes is appropriate, because the old de Young building was damaged beyond reasonable repair by the 1989 Loma Prieta temblor.

Goldsworthy is an artist who creates artworks in the natural landscape using nature's materials to form sculptural work of deceptive simplicity, often achieving amazing feats of balance and timing in the process. Whether ephemeral, permanent or designed to age with time, Goldsworthy's works inspire quiet introspection about the beauty of the world as a living organism in a state of continuous change.[31]

He explained that he sees it as a drawing, and indeed its title, *Drawn Stone*, reflects that notion. Goldsworthy has been using the motif of cracked stone for some time. "The crack is a window, a way into the stone," he mused. [12]

The meandering portions support a discussion of rough similarity and fractals, and the zigzag can introduce the taxi metric. The pavers invite counting tasks, and the crack adds a lot of fraction problems. One can seek the principle of the crack's path and suggest alternative layouts or other goals. There is the puzzle of execution, which opens questions like how many pavers need to be cracked to get ones that work into the design. Overarching all of that is a conversation on the artist's process in manipulating nature to both simulate and reveal regular patterns. The mathematical point of view can build a sense that this crack in the ground is no accident, that it may be the best crack.



Figure 4. *Drawn Stone* by Andy Goldsworthy. De Young Museum.

6. FP abroad and at home

6.1. *Introducing FP to others*

FP has been making contacts with institutions in the Bay Area, including the American Institute of Mathematics and the Mathematical Sciences Research Institute (a cosponsor of BIRS). In spring 2008, FP participated in the second annual Julia Robinson Mathematics Festival for middle and high school students, held at Pixar Animation Studios in Emeryville. Through this activity, many people involved in math education and enrichment in Northern California were introduced to the opportunity provided by FAMSF and the Fusion Project. In addition to the Bridges 2009 papers [27, 28], we intend to submit more detailed descriptions of the material to NCTM journals JRME and MTMS.

6.2. *FP and USF*

When we realized that the material from the TAG was flowing in but we had no model for how it should flow out, Wagner proposed contacting USF's School of Education. Two phone calls later found us at the start of an ongoing conversation and collaboration with Caryl Hodges, Associate Dean for Teacher Training. She has had lengthy experience with running workshops for teachers, and she convinced us that it would be easy to incorporate FP into the fabric of USF in a way that facilitated obtaining grants to pay teachers for FP training and implementation. Not only is this reasonable and fair, it is necessary, now that California no longer requires teachers to take continuing education units, usually at their own expense in the past.

In spring 2009, FP was adopted by the USF College of Arts and Sciences as a research and education program. In addition:

- The Israel Family Foundation has made a 2009 research grant to FP;
- Jeff Buckwalter, Associate Professor of Computer Science and Director of the two schools'

Dual Degree in Teacher Preparation program, has joined the FP effort; he, Hodges, and Wells are co-principal investigators on FP;

- The TAG members have agreed to act as instructors for the first rounds of the proposed workshops;
- We all realize that it is essential to have classroom tracking and followup sessions, and we foresee documenting the performance of FP materials and methods in classrooms;
- FAMSF is supportive of an active role played by USF in sponsoring FP and has offered the facilities at the de Young Museum for the workshops and followup days; in addition, they have offered Get Smart with Art [11] binders for the instructional materials;
- All parties are enthusiastic about using USF students and local peer teachers in FP-powered classrooms;
- We have a list of schools and teachers interested in such workshops;
- And perhaps most important, there is consonance between the Fusion Project and the USF Mission and Core Values [24], recognizing specifically that
 - A. Learning is a humanizing, social activity rather than a competitive exercise;
 - B. Diversity of perspectives, experiences, and traditions are essential components of a quality education in our global context;
 - C. We foster a culture of service that respects and promotes the dignity of every person.

In the months to come, FP will continue compiling draft instructional materials, which will be refined through using them in initial workshops. The TAG and the workshop participants are the best gauge of when they are complete enough for wider publication.

6.3. *FP extended*

We are already thinking beyond the FP in the Bay area and collaboration with the de Young museum. We envision an enlarged Fusion Project concept that will:

- Identify State by State those standardized math questions most frequently answered incorrectly in that State. This will replicate the methodology used in the initial project to identify those math concepts most difficult for Bay Area students at risk to comprehend. We anticipate that discovery of this data will be successful but require effort and strategic alliances.
- Broaden the use of the term art to include animation, video games, or other media familiar to today's 7th graders. This extends the natural funding base to entertainment corporations as well as public and private sources.
- Target cities with major museums and evaluate and use their collections as we did at the de Young Museum. This will facilitate students' grasp of art in a museum setting outside the classroom.
- Use computer access to show works of art and enlarge them for the classroom, when no art museum is conveniently located near a school.

The portability or ability to adapt the program depends on having written materials that can be retargeted to different art collections, preferably by Teachers Advisory Groups at each location. Involvement in such a program must remain completely voluntary for schools and classes and be free to all students. Teachers electing to use this enhancement program would attend training workshops and would be compensated. As in the developing USF pilot program, local college students and school students would be trained to assist in the classrooms utilizing the Fusion Project. Students in these roles would also be compensated.

It is essential that the programs be assessed, with the results (positive and negative) shared freely and used to make improvements.

6.4. *The future unfolding*

Our future vision already verges on reality at USF. USF has always had inquiries about online, distant learning, including two recent ones to the School of Education. The developing USF Yale National Initiative offers innovation for school classrooms through the College of Arts and Sciences. A proposed Fine Arts Core course involving museum resources is about to be launched. USF Teacher Education and our joint Dual Degree Teacher Preparation program (especially in math) are open to testing materials and jump-starting the Fusion Project for new classroom teachers. The Fusion Project can work with all of these.

Student tours of art museums are typically reserved, hands-off, and distant. Our tours are planned to involve (polite) interaction with the art and hands-on activities with the architecture and furnishings. For example, the de Young Museum can become more like San Francisco's hands-on science museum, the Exploratorium. In fact, the de Young Museum already has a reputation of being kid-friendly [16, 17], and this congeniality will be a helpful characteristic for other FP placements or similar programs.

Finally, the extension to visual media beyond the traditional art museum (or even art and science museums like the Exploratorium) suggest opportunities at USF and far beyond. In particular, we have already had several areas of contact with Pixar Animation Studios in nearby Emeryville. As one of the authors has said explicitly (and the other agrees but never put it into words): I was taught how to pass state exams and math SATs. The same test questions were frequently repeated year after year. But this is not learning. Perhaps if the text or the teacher could have shown me the concepts visually, I would not have needed to memorize the answers. But there was no Fusion Project available to me. We think this speaks to the question of the value of art and visual media. Mathematicians and math teachers use show in a sense far removed from informed seeing. Even illustrated textbooks often do not make the pictures engaging. Artists do.

7. Conclusion

Middle School students most often see mathematics as numbers and symbols in a text, on a board, or on a test—they are expected to manipulate or calculate to get an answer. Rarely is math connected to real encounters in students' lives. The math encounters proposed by FP are designed to engage students in finding math in an unlikely place—at an art museum.

Students are asked to examine, count, measure, estimate, draw, calculate, and explain as they examine works of art and elements of architectural design at the de Young Museum. Some lessons are designed for the classroom to prepare for a visit, or to follow up on a visit. Others are designed to guide students to ask and answer questions while at the museum. If a class visit is not always possible, there are “virtual visit” lessons as alternatives to actual visits.

The topics and skills that are addressed in these encounters are all from the middle school curriculum. They were chosen because they are among those least well understood by students, as measured by standardized tests, and because they also can be found in the artwork and architectural design of the de Young Museum.

The lessons in the encounters have been developed to enhance and support the lessons found in middle school math textbooks, to engage students in a non-math environment, and to provide an interdisciplinary connection between math and art/design. They can be used in various ways, including

- (1) As a basic, somewhat interventional, approach to math topics that have probably not been well supported by learning in earlier grades;
- (2) In a solid 7th grade math survey, and usable by higher grades as well, especially in relation to the California High School Exit Examination (CAHSEE);
- (3) As an enrichment program for classes (even in higher grades) that may have solid background for the standards but wish to go beyond them.

We believe the FP approach demonstrates the BIRS workshop theme of using art to teach math. Furthermore, as Craven and Friedman have pointed out, it has applicability to other school districts with demographics and museum characteristics similar to the San Francisco Bay Area.

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