A Mandelbrot Sunset

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One of the seminal lessons that came with the emergence of chaotic dynamics and fractal mathematics was the realization that even the most seemingly complex and disorderly systems in nature might be following simple rules. This discovery brought about a major shift in perspective in applied fields committed to understanding such unruly things as weather systems, animal populations, and financial markets. It has also brought about a significant change of perspective in art.

Over the years, many artists have turned to the tools of fractal geometry to create tableaus that are most often representations of fractals arising from mathematical formulas. The resulting images are necessarily abstract because this is a quality of the fractals themselves. To be sure, abstract fractal-based art has exploited the characteristic complexity and self-similarity of fractals to great effect, with suggestive images that create a sense of wonder and expectation for the viewer. These images, however, are rarely, if ever, representational. This is odd because, generally speaking, chaotic dynamics is not an abstract endeavor. The techniques of chaos theory are routinely put to use to model the turbulent and unpredictable real world; therefore, it should also be possible to use fractal images in the creation of natural, realistic phenomena and landscapes.

The intricate structure of mathematical fractals is suggestive of the complexity of nature, but rarely does a fractal on its own communicate the beauty of nature. It is the creative artist who, by recognizing and applying these properties, can create an aesthetic work of art.

We are going to briefly explain a way of using an abstract fractal image to create an absolutely realistic painting of a natural landscape called *Sea at the Sunset*. (See figure 1.) To create a realistic illustration, it is not essential to obtain a fractal similar to our subject. Fractals do often have some primary shape, and it is possible to make changes to this shape by altering the underlying formulas and parameters of the fractal, but this is a limited tool that we will largely avoid.

The most important observation is that fractal images share many properties with nature: self-similarity, fractal dimension, complexity, chaos, simultaneous order and disorder. In a very real sense, fractals are a complex and mixed illustration of natural behavior. The idea, then, is to capture these qualities of nature contained in our fractal and apply them in the image-making process. It should be noted that there were a few artists, such as Katsushika Hokusai and Jackson Pollock, who instinctively recognized self-similarity and fractal dimension as intrinsic properties of nature and applied them in their paintings.

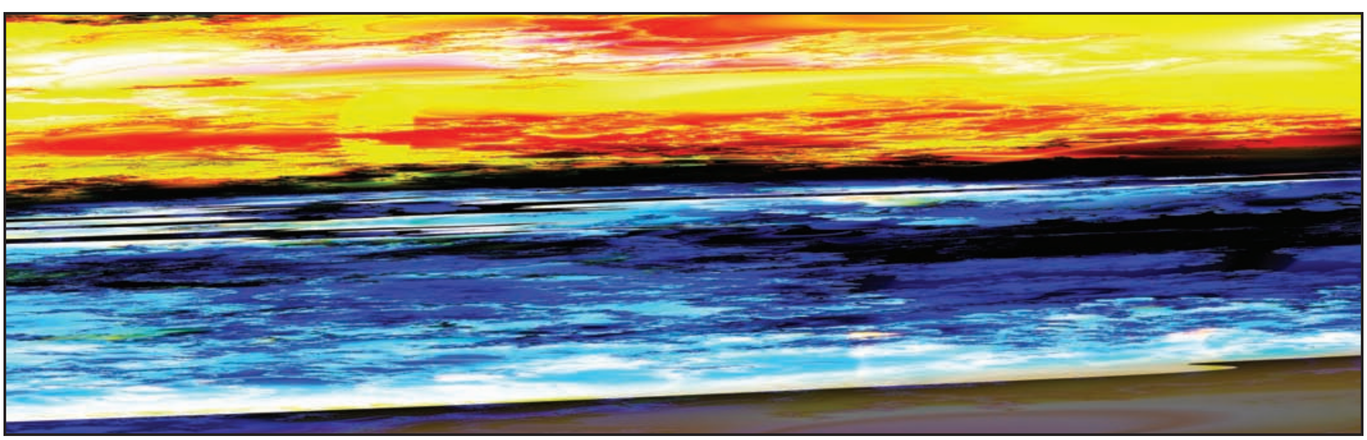

Figure 1. *Sea at the Sunset*, created by Mehrdad Garousi (2008).

Layering the Mandelbrot Set

The software package we use for our painting is called Ultra Fractal (`http://www.ultrafractal.com`), but many others would work just as well. We never use any distortion, erasing, or image-processing software. The fractal we will use is the famous Mandelbrot set, which lives in the complex plane and is created using the following algorithm:

Initial Condition: $z = 0$
Loop: $z = z^2 + c$
Bailout Criterion: $|z| \geq 2$.

The value of c in the loop equation is a complex number that corresponds to the pixel location. For a given pixel location c, the software iterates the loop, starting with $z = 0$, and checks to see if the resulting sequence stays bounded (by two.) If it does, the pixel is colored black and is in the Mandelbrot set; if the iterated sequence heads off to infinity, the point c is not in the Mandelbrot set and is given a (nonblack) color determined by the speed of divergence. Figure 2a shows the results of this process.

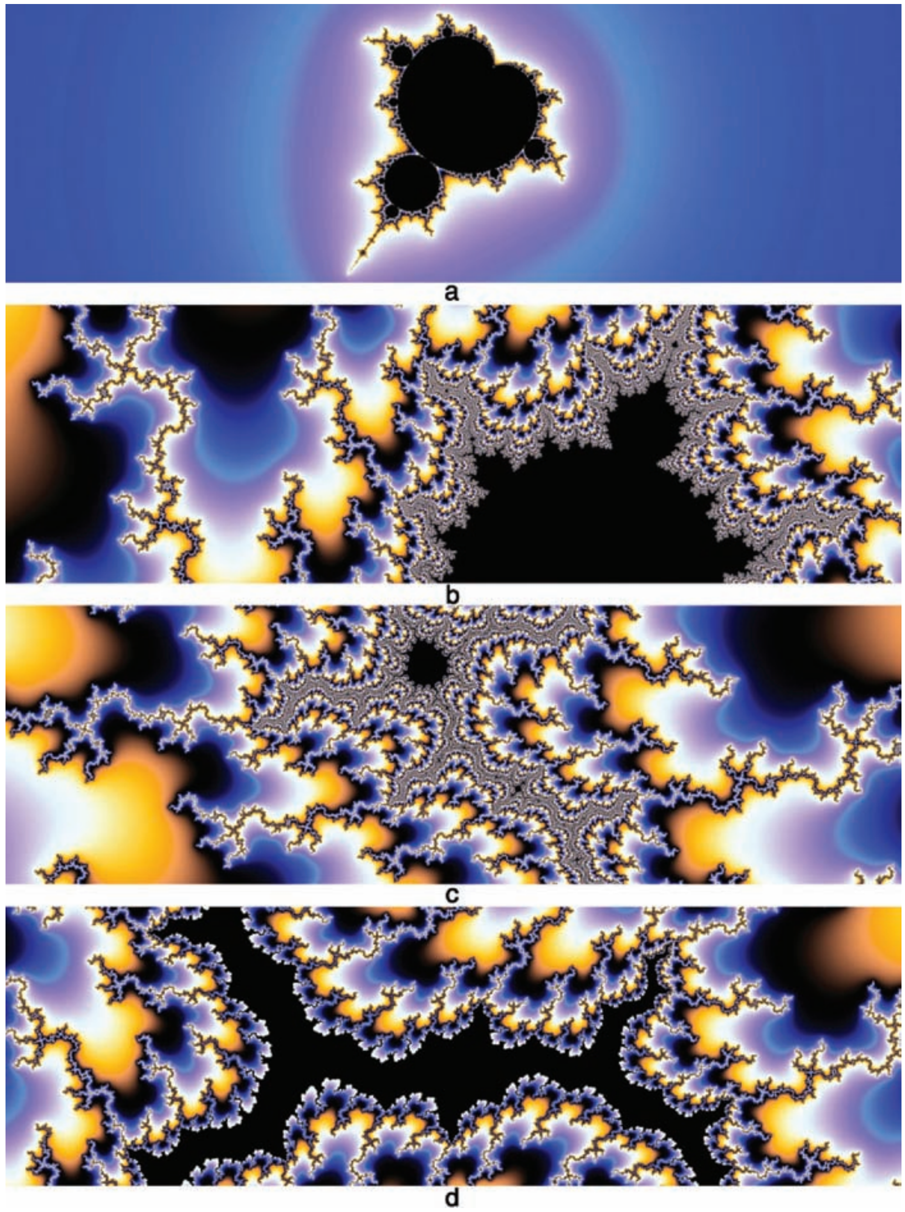

Figure 2. Process of magnification of the border of the Mandelbrot fractal. Image (a) is the entire set. The magnification factor of (b) is 5.8×10^3, (c) is 5.8×10^5, and (d) is 5.8×10^7.

The complexity in the Mandelbrot set lies at the boundary of the set (the black points) and its complement. To create our painting, we successively zoom in on a piece of the boundary to attain an image with an approximate magnification of 5.8×10^7. (See figure 2.) Ultra Fractal, like most fractal software, has an option for rotating the flat page on which the fractal is produced, which effectively changes the viewing angle but otherwise leaves the fractal unchanged. Using this feature, we adjust the viewing angle to the lower, more horizontal perspective shown in figure 3. (Note that the central, thick black line in figure 2d becomes the slim, black line that starts at the center of the right side of figure 3.)

Figure 3. The piece of the Mandelbrot boundary from figure 2d, viewed from a point near the surface.

Next, we generate five other versions of this image, each with a slight difference in zoom and transparency from the previous ones. The result of overlaying these five images on the first layer is shown in figure 4. The combination is a rich, textured image suggestive of the sea and clouds, and the strategy at this

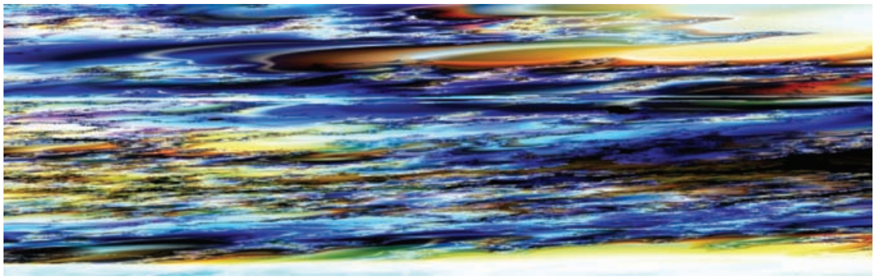

Figure 4. Result of layering six slightly modified copies of the image from figure 3.

Figure 5. The painting with 11 separated layers of the Mandelbrot set.

point is to continue layering even more images to perfect this effect. Adding copies of the same five layers as before with their colors modified and a little relocation from their previous positions yields figure 5.

Now we need to control the dispersion of the painting's colors with an eye toward differentiating the sea and sky. This time, we add five more layers of the Mandelbrot set with more blue, red, yellow, and white in the appropriate places. In most stages of this process, we don't need to change the color of the Mandelbrot fractal; rather, we just zoom in on the color that we need. For example, to make the image more yellow, we magnify a mostly yellow area of the Mandelbrot set and add it in as a new layer. The image in figure 6 consists of 16 layers of complex and disordered views taken from the Mandelbrot fractal, but by managing the superposition of different layers and by controlling the color dispersion in the whole image, we have created an evidently ordered painting representing a blue sea with a red and yellow sunset.

In contrast to the chaotic sea and sky, the round shape of the setting sun is highly regular. To create a circular form using Ultra Fractal, we can change the loop equation to

Initial Condition: $z = 0$
Loop: $z = c$
Bailout Criterion: $|z| \geq 2$,

and the resulting set of pixel values whose orbits stay bounded by two is precisely the disk of radius 2. After adding five more layers that balance the colors and display the sun in the painting, we obtain the completed landscape shown back in figure 1.

Figure 6. This stage of the painting, showing the sea and sunset, contains 16 layered images.

In the end, we have created a 21-layer landscape in which every part of the canvas is covered with detail and complexities at the finest scale. If we look at the space between two waves, we observe other, more irregular waves, and between these irregular waves, there are still more. The clouds of the painting also have this infinite property, making them visually similar to real ones in the sky.

The whole of this image is made from the Mandelbrot fractal and thus takes on these fractal properties of nature. But unlike the Mandelbrot fractal, we have an unmistakably representative image—a unique sunset painting that could not have been created on any canvas or with any camera. Its creation depends on computer assistance and on exploiting fractal geometry. It also depends on the eye of the artist. The intricate structure of mathematical fractals reflects the behavior and properties of nature, but rarely does a fractal on its own communicate the beauty of nature. It is the creative artist who, by recognizing and applying these properties, can create an aesthetic work of art. ■

About the author: *Mehrdad Garousi is a painter and photographer who has recently devoted his artistic energy to mathematical and fractal art entirely. For more information, please see* www.mehrdadart.deviantart.com.

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