The Triumph of the One

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Abstract

This note discusses some interesting coincidences involving the number 1. It is not numerology, for there is no interpretation or inferred meaning. It is not mathematics, for no deductions are feasible. It is not psychology, for nothing depends on reflection, introspection, association, or the lower mind. No special claim is made of divine intervention. But it *is* math and art.

1.

Trumping Kronecker¹: "God created 1, the rest is on us." [3] Analogously, we can construct propositional logic solely from the Sheffer stroke |.

My freshman advisor, Ken Hoffman, often lingered in the MIT dining hall writing math on the paper placemat. He held court there as well. I approached him in my second year, marveling at the axiomatic construction of the reals from the integers. He told and showed me that 1 is all one really needs.

One, One, One. One, One, One. One, One, One. There's nothing you can do that can't be done. Nothing you can sing that can't be sung. Nothing you can say but you can learn how to play the game. It's easy. Nothing you can know that isn't known. Nothing you can see that isn't shown. Nowhere you can be that isn't where you're meant to be. It's easy. All you need is one. All you need is one. All you need is one, one. One is all you need. -after The Beatles' "All You Need is Love"

¹ As Kronecker put it: "Die ganze Zahl schuf der liebe Gott, alles Übrige ist Menschenwerk" ("God created the integers, all else is the work of man.") [1]

As the famed logician Alfred Tarski's longtime and final student, I was drawn in 1967 to visit Poland, in many ways the home of modern symbolic logic. So for more than a year I left the University of California at Berkeley, becoming in order: a Stanford University exchange student in Warsaw, an arrestee in a free-speech protest march there, an icon in an April Fool's article for a Polish women's magazine, the first international member of the Warsaw Speleoklub, a lover of St. Francis, a highlight on Austrian TV while trying to cross the border north into Soviet-seized Czechoslovakia, and a translator of the Russian algebraist and logician A. I. Mal'tsev. The last led to the subject of my dissertation.

During the Alfred Tarski Centenary Conference (Warsaw, May–June 2001), I visited the new emerald library of the University of Warsaw and stood amazed below the truly monumental statues of four Lvov-Warsaw School philosophers: Twardowski, Lukasiewicz, Tarski, and Lesniewski. The last three are also renowned as mathematical logicians. Only in Poland do they elevate their philosophers—and then not on pedestals, but on eleven-meter inscribed columns at the head of a four-meter staircase! [11]



Figure 1: *The logicians among the four philosophers, Lukasiewicz, Tarski, and Lesniewski.*

Shortly after returning to Berkeley from these graduate studies in Warsaw under the direction of Andrzej Mostowski, Tarski's first student, I attended a theatrical presentation of the "Whirling Dervishes." By the secular policies of the state of Turkey, this fragment of the Mevlevi movement begun by Jelaluddin Rumi some 700 years ago functioned as a state-recognized performance troupe devoted to cultural heritage. Their smoothly turning dance is indeed beautiful, but it is not a stage show—it is a worship service.

Come, come, for you will never find a friend like Him. Where indeed is a beloved like God in all the world?

Close off your senses and open your eyes— He is the one and only treasured prize. —*Maulana Jelaluddin Rumi* [10]

The program was scheduled for November 11, or 11/11. On that day, my car's odometer turned 1111111.1. The San Francisco address for the performance was 1111 California Street.² So it was with little surprise that we counted 11 dancers and 11 musicians that evening.

The Sheffer stroke | is identified today with NAND (not-and; originally, it was NOR [9]), and (A|A)|A represents **T**, or truth. Reading this as */anandananda/ (the operator | is commutative), we hear in Sanskrit, self-referentially, the "bliss of bliss."

It's just a thing, an abstract thing!

—John Nash [8]

The time shifts forward, to the agonizing period many years later when I (but not alone³) overcame great difficulties—most of my own making—and finished my doctoral dissertation under the direction of Alfred Tarski. It had to be signed and filed by Friday, November 12. Perhaps because of failing health, or perhaps because of the legendary determination to get as much productivity out of his doctoral students as possible, he delayed, or was delayed, until the final deadline approached. Things looked tight, but not yet hopeless, when I had the realization that eleven 11/11s had passed since that evening of the Dervishes. Could it mean that this process would be completed just in the nick of time, on 11/11?

Things that are real are given and received in Silence	<i>—Meher Baba</i> [7, p. 6]
Silence is often the best thing to say.	<i>—Bene Gesserit graffito</i> [2, p. 111]

This fantasy provided support if not buoyant hope during the next ten days, but in fact Thursday, November 11, passed with no signal of intent to sign, and it looked like the eleventh and final day would pass as well. Hoping for a last-minute signature, I left the title page with his wife, Maria Tarska, on Friday morning. At 3:30 p.m., I drove to their house in desperation to see if by some chance he had rallied enough to consider signing. There appeared to be no one home.

One moment in her presence And you can forget the rest. For the girl is second best To none, son. Ooh! Sigh! Give her your attention. Do I really have to mention She's the one?

-"One" from A Chorus Line [5]

Dejected and worried by the thought that he might have had to revisit the hospital, I returned to the University of California. But there was great news from committee member Ralph McKenzie: "Maria called. Alfred just signed it!" I quickly drove back to the Tarskis; I could not beat the 5 p.m. deadline, but the administrative assistant was going to keep the office open for me. At the door, Maria handed me the envelope in which I had brought her the unsigned title page that same morning. I rushed to the car; as I reached the sidewalk, I paused to reassure myself it had his signature. It was indeed signed, and it was dated in his familiar writing—not November 12, but the day before, November 11!

² That this is a Masonic auditorium gives an additional esoteric patina.

³ Overheard mystical conversation: "Alone at last." "You are never alone." "No one does anything alone." "No one is *alone.*" "*No one* is alone." From this one might derive: "You and I are not we, but one." Friends of mine said the first five, and Meher Baba said the last [7, p. 26].

"Sufis say that real truth is always spoken with love, and that every word we speak must first pass through three gates. At the first gate we ask ourselves, 'Are these words true?' If so, we let them pass on. At the second gate we ask, 'Are they necessary?' At the last gate we ask, 'Are they kind?'"⁴

Consider the symbols T and \bot , representing true and false in propositional logic, the symbols \blacksquare and \blacklozenge , representing necessity and possibility in modal logic, and finally \cup and \cap , representing union and intersection in set theory. The last pair connote inclusion and exclusion, with the additional suggestion of pleasant and unpleasant via the theatrical mask (and popular face) icons. Thus the criteria of the three gates of speech can be symbolized by $\underline{\mathsf{TL}} \blacksquare \diamondsuit \bigcup \cap$; Gate 1 is T .



Figure 2: Three Gates.

This was the last dissertation Alfred Tarski signed. A few years ago, I saw the Mevlevi Dervishes for the second time. In the theater lobby I eagerly bought a videotape of an earlier performance recorded at the Cathedral of St. John the Divine in New York City.⁵ Only after watching it did I notice that the performance had been taped several years before on November 11, and indeed eleven 11/11s had intervened since my thesis was dated.⁶

The primary school teacher found that his student could not even begin to learn the alphabet, which in Arabic starts with alif, written |. This symbol is also the numeral one, unity. And so the student went into the world, ill prepared by schooling. He returned years later and visited his old classroom; he told his primary teacher that he had finally achieved learning. "You can write?" "Well, I have a start—I know alif!" "Please demonstrate your learning at the chalkboard." The former student went to the board, in silence drew the single stroke. The board cracked, the wall split, then fell out into the yard, opening the room to the garden. The teacher smiled. "Truly, you have learned your lesson well."⁷

⁴ From a clipping, source unknown, but this is a popular story, poem, song.

⁵ Where in Manhattan? Call it 111th Street.

⁶ It is amusing that, with no intention, my current automobile turned all 1s on 1/11, at approximately 1:11 p.m. (I missed the exact time). This article was first submitted as a reminiscence for inclusion in the Tarski Centenary Conference proceedings, but the format of that changed, and the planned section on reminiscences was abandoned. In fact, there was also only one submission; its revised $T_E X$ version was sent at 1:11 a.m., January 11, 2002, and the byte count was 10101; but this time there was some intention.

⁷ Traditional story. For an authoritative version see [4].

Alfred Tarski is renowned for his contributions to mathematical logic, most notably for the definition of truth (mathematical, that is). But his work spanned at least seven different areas of mathematics: geometry, set theory, modern algebra, universal algebra, equational logic, algebraic logic, and metamathematics (which he invented/discovered and named). His interests further included philosophy, education, linguistics, anthropology, paleontology, horticulture, arts, literature, and even computer science. His arguably greatest results were the Banach-Tarski paradox, the fixed-point theorem for complete lattices, and the decision procedure for real-closed fields. It is often acknowledged that he came very close to anticipating Gödel's incompleteness results for Peano arithmetic, and he was able to extend this trend to the undefinability of truth. That is no paradox, despite the first sentence of this paragraph. One of Tarski's characteristic patterns of work was a spiral path through the different fields; his courses and seminars usually tracked along in a grand cycle of seven or so years, onward and up. Perhaps he would find a harmony with my cyclic encounters with all these 1s. At least he would be amused.

Page 111:

There is a game we should play, And it goes like this. We hold hands and look into each other's eyes And scan each other's face Then I say. "Now, tell me a difference you see between us." And you might respond. "Hafiz! Your nose is ten times bigger than mine!" Then I would say, "Yes my dear, almost ten times!" But let's keep playing. Let's go deeper. Go deeper. For if we do, Our spirits will embrace And interweave. Our union will be so glorious That even God Will not be able to tell us apart. *There is a wonderful game* We should play with everyone And it goes like this ... *—Hafiz of Shiraz* [6]

References

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[4] Hazrat Inayat Khan, http://www.universalsufism.com/Story_of_Alif.htm (11/11/2002)

[5] E. Kleban, lyricist, A Chorus Line. 1975.

[6] D. Ladinsky, I Heard God Laughing: Renderings of Hafiz. Sufism Reoriented, Walnut Creek CA. 1996. Page <u>111</u>.

[7] I. Luck, The Silent Master, Meher Baba. Meher Baba Archives, Myrtle Beach SC. 1987.

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[10] Unknown translation of Rumi

[11] B. Wells, *Is there a nonrecursive decidable equational theory*?, Minds and Machines, Vol. 12, pp. 303–326. (Special issue on effective procedures, edited by C. E. Cleland). 2002.