

Bridging Art Museums and Middle School Math Classrooms

Benjamin Wells *
Departments of Mathematics and Computer Science
University of San Francisco, 2130 Fulton Street, San Francisco CA 94117 USA
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Philip Wagner
The Fusion Project, 736 Clipper Street, San Francisco CA 94114 USA

Abstract

The Fusion Project is a research program under development at the University of San Francisco, envisioned by Philip Wagner and directed by Benjamin Wells. With the collaboration of the Fine Arts Museums of San Francisco (and their de Young Museum), it seeks to bring art to the math classroom and math students to the art museum. The goals are to enhance existing curricula in order to improve basic and advanced skills, standards-oriented test scores, and students' interest in mathematics. Therefore, it is a quintessential Bridging project.

1. The Fusion Project: Some Background

Following the US federal program called No Child Left Behind, the performance of US public schools has been judged by standardized test scores in language and mathematics. Time and again, the authors heard that individual teachers' evaluations as well as school funding depend heavily on improvement of student performance on these tests.¹ Despite the pressure and effort, scores remain unacceptably low² in California, a state with the seventh largest gross domestic product in the world in 2007.³ San Francisco schools in particular have been below the program's pass level every year.⁴

Philip Wagner, a retired California businessman, was inspired by his perception of the crisis in math education and his own love for art to create The Fusion Project [1] (FP) as an antidote—or better, a nutritious supplement—to fight this trend in middle school math. The main idea of FP is to engage middle school students in learning some critical math concepts by discovering them displayed in artwork and architectural components of a major art museum. In our view, intervention is required to enhance interest among students who may not recognize the beauty and importance of math in their futures.

If the answer is to use art, where better to find the art than in art museums? In 2006 Phil approached the Fine Arts Museums of San Francisco (FAMSF), whose de Young Museum had recently reopened in an award-winning building in Golden Gate Park. Their Education Department saw the value of FP to students and to their museums and immediately wrote a letter of support, the first step of a unique relationship. We say unique because our queries of art museums in four countries, search on the Internet, and contacts in the Math and Art community report no similar programs.

Leonard Shlain, author of *Art and Physics* [2], referred Phil to Benjamin (Pete) Wells at the University of San Francisco (USF). Pete had hosted the controversial author for four talks in his interdisciplinary courses melding math, science, religion, and art, which he has been teaching over twenty-one years. Moreover, Pete had trained peer teachers to deliver Socratic curricula to entire math classes in 6th–10th grades for eight years under Oakland Public Schools and UC Berkeley sponsorship.

The purposes of FP. The first goal of FP is to demonstrate that some children can improve their interest and understanding in math at a critical grade level. Recognizing the increased significance and political

importance of test results, we also hope to achieve a second goal: improving student scores on the types of problems from the state’s standardized math tests that have significant miss rates.

To meet our second goal, we needed to know the math concepts that are most challenging to middle school students. One might think this information would be readily available from the school district or the state, or even be online. Not so. While the State of California releases the total scores for the math test (for example, [3]), it does not identify those math concept areas that are most frequently difficult for students, nor does it summarize test results by individual topics, much less questions. Even teachers are not privy to this apparently classified information. Before we found a solution, we got international help.

FP goes international. In 2006, we learned independently of the workshop “Innovations in Mathematics Education via the Arts,” sponsored by the Banff International Research Station for Mathematical Innovation and Discovery, to be held at the Banff Centre, Alberta, Canada in January 2007. Thirty mathematicians, educators, and artists from four countries were invited, including Pete. Pete encouraged Phil to apply, but then could not go himself. The now-invited Phil ended up being responsible for presenting the ideas of the FP to the workshop participants. Although confident as a businessman, he felt the scope of the workshop was beyond his experience. Pete assured him of a receptive audience and enriched his PowerPoint talk with museum and art images.

Phil’s obvious concern for students’ math performance⁵ made him an instant insider. FP was quickly identified as having significant potential and was chosen for review and development by a focus group, comprising Nat Friedman, Doris Schattschneider, and Stewart Craven. They recognized the opportunities in the Bay Area with its many museums of science, nature, and art, but they foresaw an even broader context for the development and distribution of FP techniques and materials. They perceived the consonance of FP and the goals of the Banff workshop. Applying their personal experience from a variety of school systems, they advised that we would not gain traction with teachers or administrators if the program were couched as reform or even enrichment. Such attitudes can be interpreted as elitist or as undermining established, even mandated, patterns. In particular, they warned against calling FP products *curricula*. Later, Doris suggested using words like *enhancement* and *encounter*, which we adopted. The group recommended concentrating on the middle school and tailoring the materials for the 7th grade, stressing that interest in math is often gained or lost at this level.

Since the workshop, the focus group participants have made vital contributions to the development of FP. We have consulted with them electronically and hosted Doris and Nat on visits to San Francisco.

The curious mascot. While at Banff, Phil’s companions discovered an ingenious toy in a gift shop. Invented by the creator of the eponymous Hoberman Sphere, it is sold as the Switch-Pitch™. It can “switch phases” when tossed in the air with a slight spin (Fig. 1). Phil bought the shop’s entire supply and later developed a steady source so we always have them on hand.



Figure 1. *Static and dynamic changes of the Switch-Pitch [4, 5]; check youtube.com*

We have shared this essential FP emblem as a gift on first contact and a “recognition token” to benefactors, rewarded wise children who helped us see math in art, stimulated an artist who speaks math to prisoners, and shared the balls with university executive officers. We always point out that the ball can be viewed as a self-dualizing tetrahedral compound, such as the stella octangula. The static shape was

already known to Neolithic Scottish stone carvers [6] (Fig. 2). Similar stone balls are a source of speculation that the Platonic solids were known well before the Greeks [7]. Although not art, this mascot fits our broader paradigm of communicating math via visually (and haptically) attractive representations.



Figure 2. Neolithic Scottish type 2 stone balls (1, 2 University of Aberdeen Museum; 3, 4 famed Towie ball, and a drawing of it) [7, 8]. Note the chirality in the stone balls as well as in the mascot.

2. FP Connects the Math and the Museum

Both FAMSF and FP felt that a strong correspondence between the art and the math was required for a successful collaboration. Shortly after the Banff workshop, the FAMSF Education Department received a grant from the Israel Family Foundation in Colorado. FAMSF then hired Pete to correlate works of art with target standards in the California 7th grade math curriculum. Two things made this feasible: his experience with aligning (matching) standards and problems, and identification of the crucial standards.

Pete had performed alignments for five years for the Center for Educational Policy Research at the University of Oregon. For example, their Standards for Success, a project of the Association of American Universities, produced a list of Knowledge and Skills for University Success [9] endorsed by 28 leading research universities. The project in which Pete participated aligned that list with state high school standards and assessments [10]. FP could reveal the art-math connections, if they were there.

Data difficulties. Identification of the crucial math standards proved to be a difficult process. Student performance data on individual test problems was virtually impossible to find. Although much in education in America, and particularly in California, currently depends on published test results, these are pooled, and actual student behavior on individual problems is zealously (maybe jealously) guarded. Apparently administrators are not eager to have that kind of detail revealed, even by averages. We were frustrated for months, but a contact through a local public TV math-tutoring show led to a teacher at a continuation school who obtained anonymous test-item score listings. Pete then aligned the test problems with the California state standards. That gave us a set of concepts to target in order to reach our goal of improving test scores. These we call the *target standards*.

Target standards. The California 7th grade content standards [11] list 46 items (including some from Mathematical Reasoning). Using the data on test performance, Pete classified ten items as primary targets and another ten as secondary. The following table summarizes these twenty, with the high targets in **bold**. Those marked with * are also designated as key standards for 7th grade in the Mathematics Framework for California Public Schools [12, p. 112]. It may be significant that only four strong targets are key standards, and another eleven key standards are not targets of either kind.

- | | |
|---------------|---|
| NS1.2* | + , × , − , / rational numbers (integers, fractions, and terminating decimals); $(a/b)^m$ |
| NS1.3 | Convert fractions to decimals and percents; use in estimations, computations, applications |
| NS1.6 | Calculate the percentage of increases and decreases of a quantity |
| NS2.2* | Add and subtract fractions by using factoring to find common denominators |
| AF1.2 | Use correct order of operations to evaluate algebraic expressions such as $3(2x + 5)^2$ |

- AF1.3*** Simplify numerical expressions by applying laws of rational numbers (assoc, etc.), and justify
- AF1.5 Represent quantitative relations graphically; interpret parts of a graph in the situation it represents
- AF3.3*** Graph linear functions; note Δy is same for given Δx ; rise/run = slope
- AF3.4*** Plot quantities whose ratios are constant (ft/in); fit line, interpret slope as ratio
- AF4.1* Solve 2-step linear equations and inequalities in one variable over the rationals; interpret
- AF4.2* Solve multistep problems involving rate-speed-distance-time and direct variation
- MG2.1** Use formulas to find perimeter and area of elementary 2- and 3-dimensional figures
- MG2.2 Estimate and compute area of complex or irregular 2- and 3-dimensional figures by decomposition
- MG2.3 Compute perimeter, surface area, volume of 3-dimensional objects obtained from rectangular solids; scale volume, surface area.
- MG3.1** Identify geometric figures; construct with compass and straightedge (altitude, midpoint, diagonal, bisectors, circle parts, etc.)
- MG3.2 Use coordinate graphs to plot simple figures; determine length, area; translated/reflected image
- MG3.6* Identify elements of 3-dimensional objects; describe skew lines, 3-plane intersections
- MR1.1** Analyze problems from relationships, relevant/irrelevant and missing information, patterns
- MR2.3** Estimate unknown quantities graphically; solve for them by using logic, arithmetic, algebra
- MR2.4** Make and test conjectures by using both inductive and deductive reasoning

Clustering art and standards into stories. In visits to the de Young Museum, Pete surveyed works from the permanent collection on display, teasing out their mathematical content. He also noted interior and exterior architectural details that would be useful to highlight. With target standards at hand, he matched artwork on his list with the relevant math standards. A 2D matrix emerged showing that most of the identified works and all the target standards (and more) were covered by each other. But how could we organize this diffuse information in a usable way? By clustering the art works and math topics, we found a natural decomposition of both into seven groups and called them Stories/Encounters. The resulting correlation convinced both parties that the de Young Museum was highly suitable for the purposes of FP. Here are the clusters (the first three are more elementary; the last, more exploratory):

- Story/Encounter 1. Counting, adding, multiplying, grouping, distributing, and estimating
- Story/Encounter 2. Fractions, percent, parts, and rhythm
- Story/Encounter 3. Lines, slopes, intercepts, equations, and ramps
- Story/Encounter 4. Edge lengths, surface area, and volume of various 3D objects
- Story/Encounter 5. Lines and planes in space, ruled surfaces, and belts
- Story/Encounter 6. Parallels, perspective, other projections
- Story/Encounter 7. Uniformity, symmetry, chaos, confusion

Museum resources. To see how the art fits into the stories, we need to review the types of art resources that FAMSF has made available to FP:

- (1) Works of art in the permanent collections—this restriction aids stability of instructional materials, and access to the art is more open (for example, nonflash photographs may be taken);
- (2) Cultural artifacts in the ethnological collections;
- (3) Site-specific works and exhibits commissioned by FAMSF;
- (4) Architectural elements (e.g., windows, walls, pavers, rooflines, staircases, furniture);
- (5) Poster-sized prints of any image in the museum's ImageBase or 4D electronic databases⁶;

Each Encounter involves all of these: original paintings and sculpture, integral features of the building or its furnishings, ethnological artifacts, or posters to be printed by FAMSF for FP classrooms. Some works on paper, including all of FAMSF's holdings of Escher's work, are most easily accessed on the Internet.

Recruiting people and materials. Pete's report [13] on alignments and stories to FAMSF gave a skeletal structure to their involvement in the project, but FP needed more detailed materials before we could begin to think about how to build the school-end of the bridge. FAMSF sent invitations to a thousand Bay Area teachers on their mail list to join the FP Teachers Advisory Group (TAG). The Israel

Family Foundation provided another grant for TAG training and consultation. From the respondents, we chose five middle school math teachers in public, private, and parochial schools. We trained this group at the de Young Museum in late 2007, covering the FP/FAMSF report and giving a tour of major works included in it. Subsequently, the TAG communicated by email with us and with each other. These teachers were expected to develop the Story outlines into teaching materials. Story assignments were negotiated. On request, Pete offered a suggested outline [14, pp. 16–17] for expanding a Story into an Encounter—that is, a teacher-ready guide of questions and explorations that could be used in class and on museum visits. The teachers have submitted two to four sets of draft instructional materials that are currently being compiled and edited. The next section gives samples of three different levels and types of draft materials [13, 14] produced by the TAG and FP.

3. Sample Story/Encounter Materials

Story/Encounter 1: Example of a core cycle. The following problems form a *core cycle*, the essential element of the structure outline mentioned above. They center around *Strontium*, a huge photo array made up of identical rectangular panels containing large balls, small balls, and some partial balls.

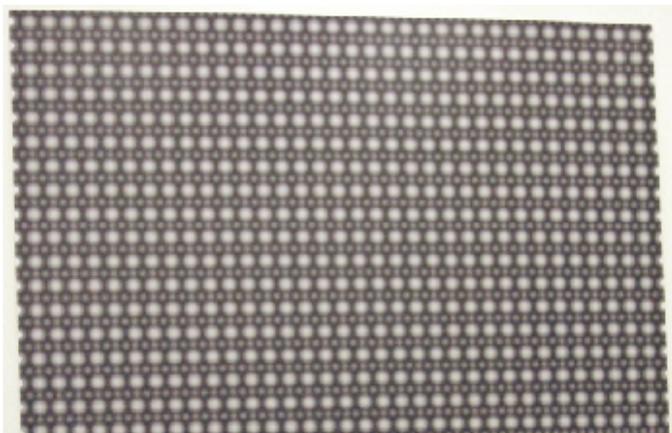


Figure 3. Top half of *Strontium* by Gerhard Richter, a FAMSF site-specific commission

Figure 4. *Strontium* at opening of the new de Young Museum—October 15, 2005 ©Art Rogers

Problem 1. Count the fuzzy balls in *Strontium* (Fig. 3).⁷ We start with the large ones.

Concept/Technique A: (1) To count a rectangular arrangement of things, count one row and multiply by the number of rows. (2) How about counting one column and multiplying by the number of columns?

Concept/Technique B: (1) To handle split half balls, count them and divide by 2. (2) Quarter balls?

Concept/Technique C: (1) Count the small balls by using the same methods, as if the large balls weren't there. (2) Count the small balls by seeing how many belong to a large ball.

Application to Problem 1, and refinement: (1) How many rows are there in the full picture? Do they each have the same number of balls? Whole balls? (2) How many columns are there? Do they each have the same number of balls? (3) So how many balls are there, by counting a row? by counting a column? (4) Is there an easier way to handle the half balls? quarter balls? small balls? (5) What about counting by panel? How many panels are there by using the methods? Are all panels the same? Is it easier or harder?

Problem 2 (driven by math). Count the number of items in a three-dimensional array. Models of this can be a stack of cocktail sugar cubes, or Montessori cubes, but the abstract problem is the focus here.

Application to Problem 2, and generalization: Use the teaching and learning from Problem 1 to generalize to three dimensions. Develop the approach of multiplying rows, columns, and layers by seeing that we can already count in each layer, and that each layer is the same size as the others.

Problem 3 (driven by an image). How many people are there in front of *Strontium* (Fig. 4)?

Application to Problem 3, and review: The method of counting the fuzzy balls does not work immediately or directly here, because people are not standing in rows and columns (but, at a school assembly, that might work). But dividing the picture into rows and columns of boxes would make it something like the panels in Problem 1. What makes this work? What keeps it from working very well?

Story/Encounter 3: introduction and worksheet fragment. (*TAG member Claire Potter with Pete and Doris*) There are many paintings in the de Young Museum that have lines or suggestions of lines that the artist has included to draw our eye to a focal point, or to have us look at a scene from a special angle. The museum also has several staircases—their steepness, or pitch, makes them easier or harder to climb. We can describe how a line appears in a painting by its slope, and can calculate this by measuring. To find the pitch of a staircase, we can imagine it replaced by a ramp, and then calculate the slope of the ramp.



Figure 5. Wayne Thiebaud, Diagonal Freeway

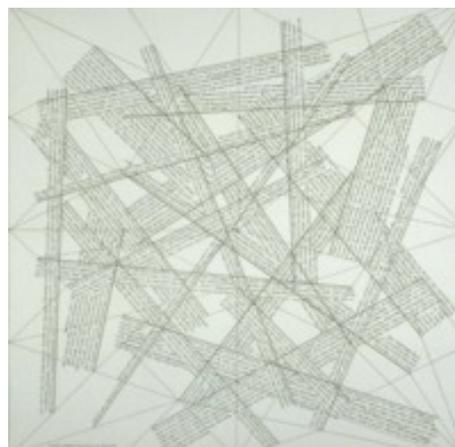


Figure 6. Sol LeWitt, Lines from Points to Points

How steep is this freeway (Fig. 5)? Notice there are several lines to consider. They appear to intersect outside the frame, somewhere below and to the left of it. Do they all intersect at the same place? How does this affect your answer?

Examine the picture by Sol LeWitt (Fig. 6). 1. Which lines [labeled in the handout] slope upwards from left to right? List them. Which labeled lines slope downward from left to right? List them.

2. In your list of upward sloping lines, which line is the steepest? Which line is the most flat? Now sort these lines according to steepness, with the most flat first, and list them in order by letter. In your list of downward sloping lines, which line is the steepest? Which line is the most flat? Now sort these lines according to steepness, with the most steep first, and list them in order by letter.

Definition: The slope of a line is the ratio of the vertical change to the horizontal change as you move from one point to another along the line. Note that as you move along the line from one point to another, if you move up, the vertical change is positive and if you move down, the vertical change is negative. The vertical change is called the *rise* and is the numerator of the ratio. As you move along the line from one point to another, if you move to the right, the horizontal change is positive and if you move to the left, the change is negative. The horizontal change is the *run*; it is the denominator of the ratio.

3. For each of the labeled line segments in the picture, use the coordinate grid superimposed on the picture to measure vertical and horizontal change between two points on the line where a grid line intersects the line. Your teacher will demonstrate how to do this. In measuring vertical and horizontal change, round your measurements to the nearest centimeter. Remember to include the sign of the change. Fill in Table 1 [to be supplied] with your measurements, and use them to calculate the slopes of the lines.

Here are some ideas for extension. These studies about slope can lead to practical representations of the Pythagorean Theorem and computing missing parts of right triangles. LeWitt's minimal, procedural art lends itself to reimplementing and elaborating. Students find it fun to make this sort of art.

Story/Encounter 4: sample tasks. (TAG member Gail Purtell)

To do in the classroom:

- Identify equilateral, isosceles, and scalene triangles; acute, obtuse, and right triangles.
- Find the area of triangles.
- Use pattern blocks to demonstrate how equilateral triangles can be tessellated various ways.
- Find the surface area of pyramids.



Figure 7. Architects Herzog and de Meuron's eucalyptus benches

Figure 8. Kane Kwei, Coffin in the Shape of a Cocoa Pod

To do at the museum:

- Notice that there are benches (Fig. 7) placed throughout the museum. Notice that each bench module covers a triangular area of floor space, and each one is made up of three smaller triangles tilted into a pyramid facing up toward the ceiling or down toward the floor.
- Identify the kind of triangle formed by the outside edges of the module. Measure the lengths of the sides of one bench module.
- Use the Pythagorean Theorem to find the height of the triangle.
- Find the floor area of the bench, i.e. the amount of floor space it covers.
- Notice the three smaller triangles that make up the pyramid within each bench module. Are the three triangles congruent? What kind of triangles are they? Measure the base and height of one.
- Compute the surface area of one bench module.

Strategies for finding the volume and surface area of the cocoa pod coffin (Fig. 8):

- To estimate the surface area, you could divide the cocoa pod into a cylinder with cones on each end. Find the surface area of each section and then add the three areas together.
- Another suggestion might be to consider each of the lengthwise strips separated by grooves that go from tip to tip. Estimate the area of each of these strips by dividing them into a rectangle with triangles at each end. Count the number of strips, then multiply to estimate the total surface area.
- The volume of the cocoa pod can be estimated by dividing it into a cylinder and two cones then finding the volume of each section. Add the volume of the three sections.

4. Next Steps at USF and Beyond

Teachers first; 7th graders next. The de Young Museum had provided a framework (literally, too; see Figs. 9, 10, which go with Story 6) and the TAG was producing material, but we had no model for the next step. A phone call to USF's School of Education led to a conversation and an ongoing collaboration with Caryl Hodges, their Associate Dean for Teacher Training. She proposed workshops for teachers and convinced us that it would be easy for FP to join Pete's College of Arts and Sciences and her School in a way that facilitated obtaining grants to pay teachers for FP training and implementation. We added Jeff Buckwalter, Associate Professor of Computer Science and Director of the two schools' Dual Degree in Teacher Preparation program. Then the de Young Museum offered facilities for the workshops and "Get Smart with Art" binders⁸ for Fusion Project materials.



Figure 9. *The Hamon Tower at the de Young Museum*

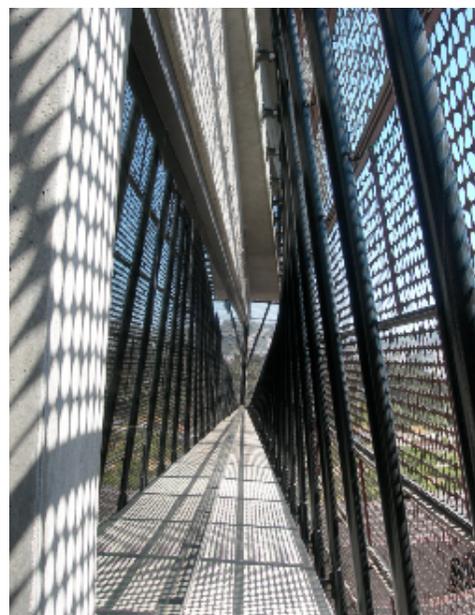


Figure 10. *Its hyperbolic paraboloid sheath*

Interest in FP at USF has grown; in April, 2009, it is expected to move from Pete's personal research project to a program of the College of Arts and Sciences. We are still developing the proper structure at USF, but progress is steady. In particular, the Deans of the College have enthusiastically endorsed FP, and the TAG members have agreed to act as instructors for the first rounds of workshops. We'll use USF students and local peer teachers in FP-active classrooms. The Israel Family Foundation has renewed their grant, and we have a list of middle schools and teachers interested in our workshops. There has been no classroom testing of FP materials yet, but we expect that to change by the next school year. Best of all, we have found great resonance of FP with the USF Mission and Core Values [15].

FP Rev. 2.0. We are already thinking beyond the FP in the Bay area and collaboration with the de Young museum. We envision an enlarged Fusion Project concept that will:

- Identify State by State those standardized math questions most frequently answered incorrectly in that State. This will replicate the methodology used in the initial project to identify those math concepts most difficult for Bay Area students at risk to comprehend. We anticipate that discovery of this data will be successful but require effort and strategic alliances.
- Broaden the use of the term *art* to include animation, video games, or other media familiar to today's 7th graders. This extends the natural funding base to entertainment corporations as well as public and private sources.
- Target cities with major museums and evaluate and use their collections as we did at the de Young Museum. This will facilitate students' grasp of art in a museum setting outside the classroom.
- Use computer access to show works of art and enlarge them for the classroom, when no art museum is conveniently located near a school.

The creation of printed guides for teachers and students will further this dissemination. The portability or ability to adapt the program depends on having guides that can be retargeted to different art collections, preferably by Teachers' Advisory Groups at each location. Involvement in such a program must remain completely voluntary for schools and classes and be free to all students. Teachers electing to use this enhancement program would attend training workshops and would be compensated. As in the developing

USF pilot program, local college students and school students would be trained to assist in the classrooms utilizing the Fusion Project. Students in these roles would also be compensated.

It is essential that the programs be assessed, with the results (positive and negative) shared freely and used to make improvements.

Math, yes, but is it art? We would like to clarify the role of art in the Fusion Project. The evaluation of art in the de Young Museum's permanent collections was based on its potential for teaching math. Most of the site-specific commissions are especially appropriate, and the entire collection is world class. But the artistic values were not assessed, nor are they directly a part of FP methodology. Art teachers were excluded from the Teachers Advisory Group for much the same reason. Our math classroom teachers are not required, expected, or recruited to teach art. That said, the authors and our advisors love and appreciate art and are stimulated by the mathematics we see in art. This will be shared by those attracted to join FP.

All consultants feel that *the heart of the Fusion Project is the art*. Some of us are collectors, some have used art in math and science courses, some have taught art, and some have been founders of the Art & Math movement. We are sensitive to the needs of museums and of art programs in schools as well, and we intend to provide structural suggestions for including an art education aspect in the Fusion Project. In particular, although the workshops will not train the math teachers in art education, they will certainly introduce them to the artistic dimensions, through guided tours by museum docents and references to art interpretation. Of course, the reason we are at the de Young Museum is because they see us as an important activity for their own art education effort. Similarly, we endorse the collaborative involvement of schools' art teachers in FP classrooms. With no emphasis on the appreciation of the art, the works in the first stories become little more than mathematical tokens to be counted and measured (but see Stories 5–7 for surface development, perspective, and procedural art, which are topics in art school courses). That is far from our goal. In fact, we believe that the interest in the mathematics that flows from the art will make the art that much more interesting and significant to the students, and we intend to help that flourish. Obviously, this attitude extends beyond the pilot program and USF.

The future unfolding. Our future vision already verges on reality at USF. USF has always had inquiries about online, distant learning, including two recent ones to the School of Education. The developing USF Yale National Initiative offers innovation for school classrooms through the College of Arts and Sciences. A proposed Fine Arts Core course involving museum resources is about to be launched. USF Teacher Education and our joint Dual Degree Teacher Preparation program (especially in math) are open to testing materials and jump-starting the Fusion Project for new classroom teachers. The Fusion Project can work with all of these.

Student tours of art museums are typically reserved, hands-off, and distant. Our tours are planned to involve (polite) interaction with the art and hands-on activities with the architecture and furnishings. For example, the de Young Museum can become more like San Francisco's hands-on science museum, the Exploratorium. In fact, the de Young Museum already has a reputation of being kid-friendly [16, 17], and this congeniality will be a helpful characteristic for other FP placements or similar programs.

Finally, the extension to visual media beyond the traditional art museum (or even art and science museums like the Exploratorium) suggest opportunities at USF and far beyond. In particular, we have already had several areas of contact with Pixar Animation Studios in nearby Emeryville. As one of the authors has said explicitly (and the other agrees but never put it into words): "I was taught how to pass state exams and math SATs. The same test questions were frequently repeated year after year. But this is not 'learning.' Perhaps if the text or the teacher could have *shown* me the concepts visually, I would not have needed to memorize the answers. But there was no Fusion Project available to me." We think this speaks to the question of the value of art and visual media. Mathematicians—and math teachers—use "show" in a sense far removed from informed seeing. Even illustrated textbooks often do not make the pictures engaging.⁹ Artists do.

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¹ Interestingly, the tests have no direct consequence for middle school students. Not until the California High School Exit Examination, first given in the 10th grade, is an individual student's score of personal importance. But receiving a diploma requires a passing score on this exit exam, which is based on *middle school math standards*.

² See [3] for 2008, and see previous Standardized Testing and Reporting (STAR) summaries for earlier years.

³ According to the International Monetary Fund and World Bank (for countries' Gross Domestic Product) [18] and the US Bureau of Economic Analysis [19], California's 2007 Gross State Product of $\$1.8 \times 10^9$ would have put the state seventh in the list of countries by GDP, between Italy and Spain.

⁴ According to a September 2008 San Francisco Unified School District press release [20], "SFUSD remains under NCLB program improvement status."

⁵ There was objective evidence for the concern. At the time of Phil's talk, the most recent STAR summary showed that "In math, 40 percent of students scored at grade level or beyond this year." [21]

⁶ ImageBase has public online access: <http://www.famsf.org/fam/about/imagebase>. 4D is a broader high-resolution image database internal to FAMSF.

⁷ This is not an easy task. The photographic assembly of 130 identical panels is gigantic, and approaching it can be disorienting, because the image is deliberately fuzzy.

⁸ Get Smart with Art @ the de Young: <http://www.famsf.org/fam/education/subpage.asp?subpagekey=19>

⁹ A strong exception to this is [22], which Pete has used as a textbook in eight courses emphasizing math and art.