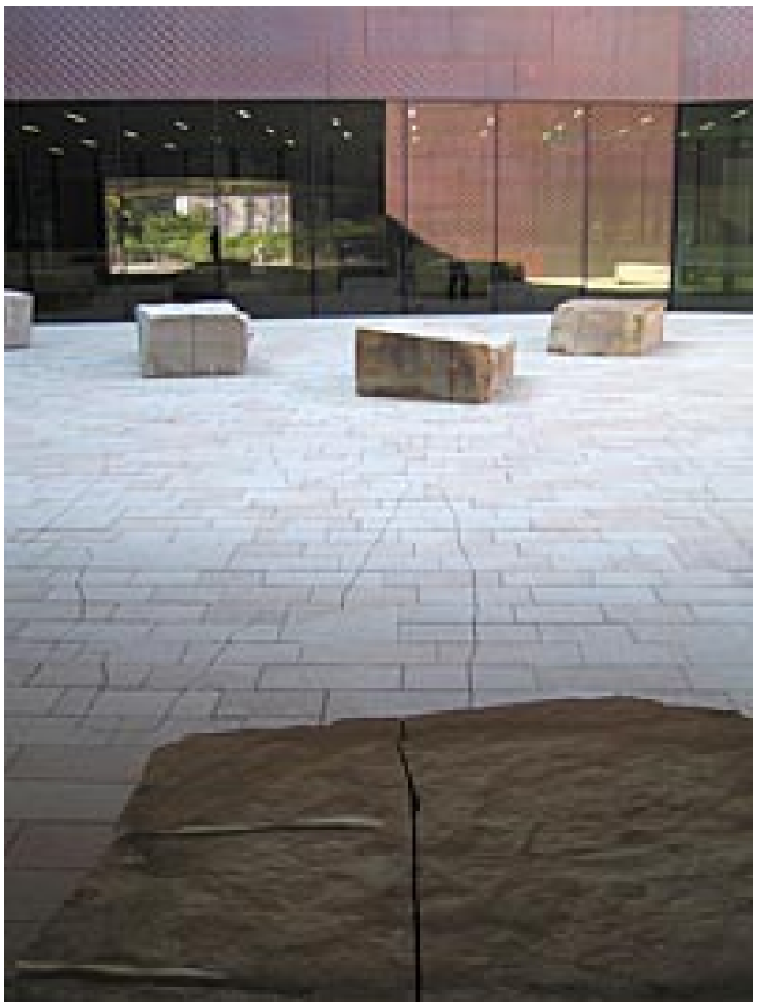
Math Stories at the de Young Museum

FAMSF and The Fusion Project



Seven treatments by Benjamin Wells

Department of Mathematics
University of San Francisco

Math Stories at the de Young Museum

FAMSF and The Fusion Project

Seven treatments by Benjamin Wells
Department of Mathematics
University of San Francisco

Summary

This report gives the results of analyzing two sets of test data and aligning these with California 7th grade math standards. The results yield "target" standards, identified by low test results. Artworks at the de Young Museum were surveyed for mathematical/pedagogical content and impact. These were then aligned with the target and near-target standards. The artworks identified as having the most relevance were organized into seven classes according to seven stories. This allowed coverage of all target standards and other useful ones. The final result is a broad proposal for math content connecting the classroom and the de Young Museum. It can serve as a curriculum guide for development of modules related to the target standards and of explorations for more advanced students and classes. The first three stories are more elementary and cover more basic standards. The last three are more exploratory and qualitative. It is envisioned that three levels of study guides could emerge:

(1) a basic, somewhat interventional, approach to topics that have probably not been well supported by learning in earlier grades;
(2) a solid 7th grade math survey usable by higher grades as well, especially in relation to the California High School Exit Examination (CAHSEE);
(3) an enrichment program appealing to higher grades as well and to classrooms that may have solid background for the standards but wish to go beyond them.

Seventh Grade Standards for Mathematics

These 46 standards are summarized from CAHSEE documents. They include 40 seventh grade standards from STAR documents plus 6 additional Mathematical Reasoning standards. They are coded NS for Number Sense, AF for Algebra and Functions, MG for Measurement & Geometry, PS for Probability and Statistics (note that no 7th grade standard involves probability), and MR for Mathematical Reasoning. The target standards were identified as those involved in the most missed questions from sample test data. The high target standards were those that appeared most often in missed questions and are labeled by bold type. In addition, * denotes <u>*key standards*</u> (Mathematics Framework for California Public Schools).

All standards

1	NS1.1	Read, write, compare, approx.: rational numbers in scientific notation
2	NS1.2*	+,*,-,/ rational numbers (ints, fracs, & term dec; (a/b)^m
3	NS1.3	Convert fracs to decs and %s; use in est, comp, applics
4	NS1.4*	Differentiate between rational and irrational numbers.
5	NS1.5*	Know every rati is either a terminating or repeating dec; term->rat
6	NS1.6	Calculate the percentage of increases and decreases of a quantity.
7	NS1.7*	Solve probs on discount, markup, commission, profit; s & c interest

8	NS2.1	Understand negative whole-number exps. *,/ exp w/comm base
9	NS2.2*	Add and subtract fractions by factoring to find common denoms.
10	NS2.3*	Multiply, divide, and simplify rational numbers by using exp rules.
11	NS2.4	Power/root of square ints; other roots between 2 ints.
12	NS2.5*	abs val, distance on number line; calc abs val
13	AF1.1	Use var, ops to write expr, eqn, ineq, sys eqn/ineq to verbal desc
14	AF1.2	Use order of ops to eval alg exprns such as 3(2x + 5)^2.
15	AF1.3*	Simplify num exprs by applying laws of ratl numrs (assoc ++)
16	AF1.4	Use algebraic terminology (e.g., va, eqn, term) correctly.
17	AF1.5	Represent quant rels graphically; interpret part of graph
18	AF2.1	Interpt +int powers as repeated *, -int as /; simp, eval exprs w/exps.
19	AF2.2	*, / monomials; take powers and whole roots.
20	AF3.1	Graph functions of the form y = nx^2 and y = nx^3; solve problems.
21	AF3.2	Plot 3D vols as fcn of edge length, base edge length.
22	AF3.3*	Graph linear fcns; note Δy is same for given Δx; rise/run = slope.
23	AF3.4*	Plot quants whose ratios are constant (ft/in); fit line, interpret slope.
24	AF4.1*	Solve 2-step lin eqns & inequalities in 1 var over the rats; interp.
25	AF4.2*	Solve multistep problems involving rdt and dir variation.
26	MG1.1	Cf. & convert wts, caps, geom measures, times, and temps
27	MG1.2	Construct and read drawings and models made to scale.
28	MG1.3*	Use measures expressed as rates & prods; use dim anal to check.
29	MG2.1	Use fmls to find perim & area of 2-3D: tri, quadril, circ, prisms, cyls
30	MG2.2	Estim & compute area of complex 2-3D figs by decomp.
31	MG2.3	Comp perim, sa, vol of 3D objs from rect solids; scaling vol, sa.
32	MG2.4	Relate the scale changes in measurement to units and conv.
33	MG3.1	Identify, construct c+se geom figs (alts, mp, diag, bisects, circs)
34	MG3.2	Coord graphs to plot simple figs; detmn ln, area; trans/refl image
35	MG3.3*	Pyth thm and converse; appls; exper verif by measuring.
36	MG3.4*	Conditions for congruence (general); meaning for sides, angles
37	MG3.6*	Elems of 3D objs; skew lines; 3-plane intersections
38	PS1.1	Var disp of data sets; stem&leaf; box&whisker; cf. 2 datasets
39	PS1.2	2 num var on a scatterplot; informal descrp of dist and reln.
40	PS1.3*	Know, comp. min, LQ, median, UQ, max
41	MR1.1	Anal. probs from relns, rel/irrel, missing info, patterns
42	MR1.2	Formulate & justify math conjectures based on a general descrp
43	MR2.1	Use estimation to verify the reasonableness of calculated results.
44	MR2.3	Estimate unknowns graphically; solve by using logic, arith, alg.
45	MR2.4	Make & test conjectures using both inductive & deductive reasoning.
46	MR3.3	Generalize results and strats and apply in novel problem sits.

Target standards; all of these are addressed in one or more stories

2	**NS1.2***	**+,*,-,/ rational numbers (ints, fracs, & term dec); (a/b)^m**
3	NS1.3	Convert fracs to decs and %s; use in est, comp, applics
6	NS1.6	Calculate the percentage of increases and decreases of a quantity.
9	NS2.2*	Add and subtract fractions by factoring to find common denoms.
14	**AF1.2**	**Use order of ops to eval alg exprns such as 3(2x + 5)^2.**
15	**AF1.3***	**Simplify num exprs by applying laws of ratl numrs (assoc ++)**
17	AF1.5	Represent quant rels graphically; interpret part of graph
22	**AF3.3***	**Graph linear fcns; note Δy is same for given Δx; rise/run = slope.**

23 AF3.4* Plot quants whose ratios are constant (ft/in); fit line, interpret slope.
24 AF4.1* Solve 2-step lin eqns & inequalities in 1 var over the rats; interp.
25 AF4.2* Solve multistep problems involving rdt and dir variation.
29 MG2.1 Use fmls to find perim & area of 2-3D: tri, quadril, circ, prisms, cyls
30 MG2.2 Estim & compute area of complex 2-3D figs by decomp.
31 MG2.3 Comp perim, sa, vol of 3D objs from rect solids; scaling vol, sa.
33 MG3.1 Identify, construct c+se geom figs (alts, mp, diag, bisects, circs)
34 MG3.2 Coord graphs to plot simple figs; detmn ln, area; trans/refl image
37 MG3.6* Elems of 3D objs; skew lines; 3-plane intersections
41 MR1.1 Anal. probs from relns, rel/irrel, missing info, patterns
44 MR2.3 Estimate unknowns graphically; solve by using logic, arith, alg.
45 MR2.4 Make & test conjectures using both inductive & deductive reasoning.

Additional standards involved in the stories or their extensions (those close to being target standards are italicized):
7 NS1.7 Solve probs on discount, markup, commission, profit; s & c interest*
13 AF1.1 Use var, ops to write expr, eqn, ineq, sys eqn/ineq to verbal desc
16 AF1.4 Use algebraic terminology (e.g., va, eqn, term) correctly.
21 AF3.2 Plot 3D vols as fcn of edge length, base edge length.
27 MG1.2 Construct and read drawings and models made to scale.
28 MG1.3* Use measures expressed as rates & prods; use dim anal to check.
35 MG3.3* Pyth thm and converse; appls; exper verif by measuring.
36 MG3.4* Conditions for congruence (general); meaning for sides, angles
38 PS1.1 Var disp of data sets; stem&leaf; box&whisker; cf. 2 datasets
40 PS1.3 Know, comp. min, LQ, median, UQ, max*
42 MR1.2 Formulate & justify math conjectures based on a general descrp
43 MR2.1 Use estimation to verify the reasonableness of calculated results.
46 MR3.3 Generalize results and strats and apply in novel problem sits.

Uninvolved standards that are close to being target standards:
10 NS2.3* Multiply, divide, and simplify rational numbers by using exp rules.
11 NS2.4 Power/root of square ints; other roots between 2 ints.
12 NS2.5* abs val, distance on number line; calc abs val
18 AF2.1 Interpt +int powers as repeated *, -int as /; simp, eval exprs w/exps.
19 AF2.2 *, / monomials; take powers and whole roots.

Remaining uninvolved standards:
1 NS1.1 Read, write, compare, approx.: rational numbers in scientific notation
4 NS1.4* Differentiate between rational and irrational numbers.
5 NS1.5* Know every rati is either a terminating or repeating dec; term->rat
8 NS2.1 Understand negative whole-number exps. *,/ exp w/comm base
20 AF3.1 Graph functions of the form y = nx^2 and y = nx^3; solve problems.
26 MG1.1 Cf. & convert wts, caps, geom measures, times, and temps
32 MG2.4 Relate the scale changes in measurement to units and conv.
39 PS1.2 2 num var on a scatterplot; informal descrp of dist and reln.

Because Mathematical Reasoning standards are overarching and not independent of content, there are no problems that address them directly. In a similar way, specific applications for them are not suggested below. They will be apparent from the questions asked.

NOTES:

1. Some of the standards focus on graphical representation. That may mean on graph paper, but it is also interpreted here as meaning visual representation.

2. Artworks for each story are grouped in Main and Additional categories. There may be others that should be added, but in most cases, both groups can be reduced without harm.

3. The stories are ordered by the numerical order of the standards they principally address, not by their intrinsic interest (which is probably the reverse order).

4. Calling the chapters here stories is a bit of misnomer, because there is no plot, only development. But stories could be added. There is also a vague connection with the stories of the tower, but seven was the right number here instead of nine.

5. It is hard to imagine motivating every student with this material, so the best approach is to use teams and small groups. This will work well in the classroom but be particularly useful onsite.

6. Visiting classes should plan strategies in advance to maximize coverage onsite. Two or three stories per visit seems likely. On the other hand, it may prove helpful to develop the curriculum so all 7 stories are used, with many fewer works involved and a focus only on the target standards. In that case, an appendix could introduce more works and more standards.

7. Equipment needed by each student or pair-team: gridpaper pad and pencil, tape measure, ring & string (a snapring on a string to determine a vertical for measurement)

8. Calculators: these should be optional; simple calculations should be done on paper. Roots of nonsquare integers will require a calculator, although some problems might address estimation without a calculator.

9. With socially and politically loaded works (such as *The Burning of LA*, *Anti-Mass* and *The Meat Market*), the teacher/guide must be certain not to marginalize the content in favor of the mathematical structure of the composition.

10. The staircases (even the missing spiral one) can be replaced by ones at school till the field trip to the de Young Museum.

11. The more evanescent works (in the Saxe Gallery for instance) are listed near the end, and prints from the ImageBase and 4D (LeWitt, Escher, perspective) are listed last. The prints would only be used at school, or posters might be provided at the museum for access by teachers or guides.

These are the artworks at the de Young Museum named in this report

	Sorted by name **Artwork/architecture**	**Artist/source**	**Location**
1	3 Gems	Turrell	Garden
2	3 Machines	Thiebaud	1st floor
3	Anthropomorphic board	Dyak	2nd floor
4	Anti-Mass	Parker	1st floor
5	Apples	Kraitzes	Garden
6	Arc	Torii	Saxe
7	Asawa wire sculptures	Asawa	Tower Foyer
8	Aurora 2006	Honda	Saxe
9	Belvedere	Escher	ImageBase
10	benches	architecture	Atrium
11	Burning of LA	Petlin	1st floor
12	Button blanket	Haida	1st floor
13	Carved mammoth tusk	Alaska	Americas
14	Cocoa pod coffin	Kwei	2nd floor
15	Collection Descending	Bollinger	Atrium
16	Conservation Chair	Cederquist	Saxe
17	copper skin and exterior walls	architecture	Exterior
18	Diagonal Freeway	Thiebaud	1st floor
19	Dinner for Threshers	Wood	1st floor
20	Drawn Stone	Goldsworthy	Court
21	Feather tunic	Peru	Americas
22	From the Garden of Chateau	DeMuth	1st floor
23	Gambling basket	Modoc	1st floor
24	Geometric Figures within Geom. Figs.	LeWitt 25	ImageBase
25	girandole mirror	American	2nd floor
26	glass panel walls	architecture	Atrium
27	Haida bentwood box	Haida	Americas
28	Hamon Tower	architecture	Tower
29	Hovor II	El Anatsui	Upper Hall
30	Igbo door	Igbo	2nd floor
31	Lines from Point to Point, pl. 4	LeWitt 458	ImageBase
32	Meat Market	Herms	1st floor
33	Mill Room	Ault	1st floor
34	Model: Total Reflective Abstraction	McElheny	1st floor
35	Oshun fan	Africa	Africa
36	Other World	Escher	ImageBase
37	pavers in courtyards	architecture	Courts
38	perspective diagrams	French	4D
39	perspective plates	French	4D
40	Peruvian vessel	Peru	1st floor
41	Piazzoni murals and room	Piazzoni	Piazzoni
42	Pierced Monolith	Hepworth	Court
43	Print Gallery	Escher	ImageBase
44	Prometheus Bound	Cole	2nd floor
45	Raceme	Clayman	Entrance

46	Rainy Season in the Tropics	Church	2nd floor
47	Rapids Canyon	Higby	2nd floor
48	Ritual oil dish	Wuvulu	Oceania
49	roofs and pentagonal court	architecture	2nd floor
50	Sewing table	Shakers	2nd floor
51	Shadow Frieze	Cook	Saxe
52	Squares w/ A Diff. Line Direction..., pl 10	LeWitt 122	ImageBase
53	staircases	architecture	Atrium
54	Storage basket, Cooking basket	Dick	Americas
55	Strontium	Richter	Atrium
56	Study of Architecture in Florence	Sargent	2nd floor
57	Terrace overhangs/Mondrianic shadows	arcjh	2nd floor
58	The Limited	Marsh	1st floor
59	Union Rave	Gursky	Atrium
60	Untitled	Shapiro	Garden
61	UW84DC #2	Deacon	Atrium
62	Whirlpools	Escher	ImageBase

The Opening of the New de Young Museum in Wilsey Court—October 15, 2005 ©Art Rogers

Story 1. Counting, adding, multiplying, grouping, distributing, and guessing

You are facing *Strontium*, a wall filled with identical fuzzy balls. Your math teacher asks you how many there are. If you stand back too far, you cannot keep track; if you stand close, the fuzziness becomes confusing. So the question of how many begins to be more important, more challenging, if only to get your math teacher off your back and stop being dizzy. Happily, the artist, Gerhard Richter, has broken the wall into 130 identical panels. That should make counting easier. The number of people in *Union Rave* is much harder, but the ideas from counting *Strontium* can make it easier.

Math questions related to artworks

Problems, skills, and techniques exercised by answering the questions: multiplication, fractions, associative and distributive laws, averaging data.

Principal artworks:

Strontium

A. Find a panel and count the large balls in it. So how many balls? Now include the half-balls on the edges.
B. How many panels across?
C. How many panels up?
D. What are all the arrangements you could have? (factor 130)
E. How about counting the small balls? Separately? Together with the large balls?

Collection Descending

F. How many images appear on the wall?
G. How many are there at a time? How many wall-fulls?
H. It is easier to count the number of columns, but the number in a column is difficult. Maybe several people could count some column and the answers could be charted. Then the max, min, median, LQ, UQ could be charted with a box and whisker plot. Even without that, an average of several counts seems to be required. (Someone may get the idea of freezing it with a cellphone camera, but this should be avoided till after data collection.)

Union Rave

I. How many people appear in the photo? Although it is not anything like the orderly Strontium, an estimate can be made by counting the number in a block and then counting the blocks that would cover most of the people.

Hovor II

J. Counting how many foil seals were used to construct this sculpture would be difficult. But what you learn from the the previous art can help in a limited area, and that can give an estimate of the whole. One website says there are hundreds. That is incorrect if there are thousands--what do you think?

pavers in courtyards, galleries, and halls

K. Almost everywhere in the de Young Museum, you are walking on tiles set in different arrangements. Some are regular (same size and shape) and are easier to count. See how the methods from the art above make these easier to count. For some areas, it is enough to recognize that the tiles are so irregular that estimates are the best than can be done, unless every single tile is marked and counted. At least it's easier than counting those fuzzy balls.

Additional artworks:

The Limited Count ties, cars, windows.
Oshun fan Count beads, zigzags, triangles
Anti-Mass Count wires, charcoal pieces
3 Gems Count pavers ring by ring. Explain the answers.
Apples It is easy to count the apples. Why?
Burning of LA Count the horizontal strips, people.
Asawa wire sculptures Find a simple way to count the branches.
3 Machines If you know how many gumballs are in one machine, what comes next?
Sewing table Count the faces, edges, and corners, and the legs. Don't touch!

Extensions and explorations

A. What is the smallest rectangular piece that could build *Strontium* by tiling without rotation?
B. What is the smallest rectangular piece that could build *Strontium* by tiling *with* rotation?
C. Why is it hard to see this work up close?
D. In *The Limited*, counting the cottonlike "cloud balls" is much harder. The edges are uncertain and there are balls inside the smoke plume. (also related to Stories 4 and 7)

E. In *3 Machines*, the number of balls in one machine that the artist actually painted is not difficult to count, but what has that to do with the number balls in that machine in three dimensions? (also related to Stories 4 and 7)

F. The *Sewing table* has to balance four sides with three legs. Why is this a problem? What arrangements make more sense for the use of the table? Why are there four sides? three legs?

Standards and artworks summary

Target standards principally addressed by this story:

2	**NS1.2***	**+,*,-,/ rational numbers (ints, fracs, & term dec); (a/b)^m**
14	**AF1.2**	**Use order of ops to eval alg exprns such as 3(2x + 5)^2.**
15	**AF1.3***	**Simplify num exprs by applying laws of ratl numrs (assoc ++)**

Other target standards addressed by this story:

6	NS1.6	Calculate the percentage of increases and decreases of a quantity.
9	NS2.2*	Add and subtract fractions by factoring to find common denoms.
17	AF1.5	Represent quant rels graphically; interpret part of graph
25	AF4.2*	Solve multistep problems involving rdt and dir variation.
41	**MR1.1**	**Anal. probs from relns, rel/irrel, missing info, patterns**
44	**MR2.3**	**Estimate unknowns graphically; solve by using logic, arith, alg.**
45	**MR2.4**	**Make & test conjectures using both inductive & deductive reasoning.**

Additional standards involved in this story or its extensions:

13	AF1.1	Use var, ops to write expr, eqn, ineq, sys eqn/ineq to verbal desc
27	MG1.2	Construct and read drawings and models made to scale.
38	PS1.1	Var disp of data sets; stem&leaf; box&whisker; cf. 2 datasets
40	PS1.3*	Know, comp. min, LQ, median, UQ, max
46	MR3.3	Generalize results and strats and apply in novel problem sits.

Main Works:

Strontium
Collection Descending
Union Rave
Hovor II
pavers in courtyards, galleries, and halls

Additional Works:

The Limited
Oshun fan
Anti-Mass
3 Gems
Apples
Burning of LA
Asawa wire sculptures
3 Machines
Sewing table

Story 2. Fractions, percent, parts, and rhythm

You enter the gallery that is the new home of the *Piazzoni Murals*, set into identical alcoves. The Land side shows scenes inland scenes, the Sea side gives coast views. It is hard to grasp all of them, and small groups from your math class gather before different scenes. The teacher notices that 1/6 of the class is looking at 1/4 of the murals and these two numbers are the same. Luckily the number of mural wall alcoves is 6 + 6 = 12, and that makes for easy fractions. Harder fractions come from trying to break down a mural into the portions of sky, water, and land shown. When art has parts, there arises a sense of rhythm through repetition.

Math questions related to artworks

Problems, skills, and techniques exercised by answering the questions: arithmetic (and maybe some algebra) with fractions, percent, conversion of fractions to percent, easy addition of fractions with unlike denominators.

Principal artworks:

Piazzoni Murals gallery

A. If 1/6 of the class equals 1/4 of the mural niches, then how many students are in the class?
B. One-sixth of your class would be how many niches? what fraction? what percent?
C. If we add on the niches next to the doors, what percent increase is this? Fractions with denominators 7 and 14 now arise too.
D. How many different ways are there to pick 1/3 of the murals? 1/4 of them? 1/6 of them?
E. Fraction problems based on the ceiling slots and lights can work as well.
F. There are 7 window panels which can be combined with the 14 total niches, etc.

3 Gems

G. Fraction problems based on the benches (9), the exterior wall segments (6), the pavers (24 in each ring) etc.

Dinner for Threshers

H. Fractions based on 12 again, or 6, 7, 14 by using one side of the table or the head and foot. Then 16 or 17 using the wait-staff.

Burning of LA

I. Fractions based on 13 strips in each of four panels. Provides parallel to weeks in a year. Also the figures are worth counting. There is only one white figure (or at most a few), so percent could be interesting. How about the fraction of strips with or without smoke?

glass panel walls in Atrium

J. There are walls of 2, 4, 10, 13, 18, 31 window panels.

Additional artworks:

Apples Fractions based on 12 again.
Model for Total Reflective Abstraction There are 24 objects, so interesting fractions are available.
Strontium Many of the counting tasks lead directly to fractions and percent questions.
pavers in courtyards As in *Strontium*, but here the irregular tiles prompt estimates of range, percent differences in tile length, etc.
Oshun fan Grouped into 4 segments, there are also numerous subsections that lead to fractions.
Pierced Monolith The simple questions here are: what fraction/percent is blue? is hole? No easy answers, but modeling on grid paper is one approach.
The Meat Market Use fractions to interpolate prices for the two unpriced items, based on the desirability of the meat.

Extensions and explorations

A. The *Piazzoni Murals* have land, sea, sky depicted in them. What fraction or percent does each occupy? How do these fractions change from mural to mural?
B. How do the pavers increase in size on the floor of *3 Gems*?
C. In *The Meat Market*, base prices on certain wholesale prices, put certain items on sale, and ask questions about discount, markup.
D. In the *Model for Total Reflective Abstraction*, there numerous explorations of interreflection to inspire multiplication and addition.

Standards and artworks summary

Target standards principally addressed by this story:

2	**NS1.2***	**+,*,-,/ rational numbers (ints, fracs, & term dec); (a/b)^m**
3	NS1.3	Convert fracs to decs and %s; use in est, comp, applics
6	NS1.6	Calculate the percentage of increases and decreases of a quantity.
9	NS2.2*	Add and subtract fractions by factoring to find common denoms.

Other target standards addressed by this story:

15	**AF1.3***	**Simplify num exprs by applying laws of ratl numrs (assoc ++)**
17	AF1.5	Represent quant rels graphically; interpret part of graph
25	AF4.2*	Solve multistep problems involving rdt and dir variation.
41	**MR1.1**	**Anal. probs from relns, rel/irrel, missing info, patterns**
44	**MR2.3**	**Estimate unknowns graphically; solve by using logic, arith, alg.**
45	**MR2.4**	**Make & test conjectures using both inductive & deductive reasoning.**

Additional standards involved in this story or its extensions:

7	NS1.7*	Solve probs on discount, markup, commission, profit; s & c interest
27	MG1.2	Construct and read drawings and models made to scale.
43	MR2.1	Use estimation to verify the reasonableness of calculated results.
46	MR3.3	Generalize results and strats and apply in novel problem sits.

Main Works:

Piazzoni murals and room
3 Gems
Dinner for Threshers
Burning of LA
glass panel walls

Additional Works:

Apples
Model for Total Reflective Abstraction
Strontium
pavers in courtyards
Oshun fan
Pierced Monolith
The Meat Market

Story 3. Lines, slope, intercept, equations, and ramping up

There are many staircases, 2 down along the fern garden wall, one down in the store, and two up to the second floor galleries, and others you cannot use in the tower. Some are easier to walk up than others, because their steepness or pitch is less (or, maybe that makes them harder). To understand the pitch of a staircase, we have to imagine it replaced by a ramp, and then calculate the slope of the ramp. But which slope? For example, the main staircase to the second floor is steeper along one edge than the other, and neither is as steep as at the center bannister.

Math questions related to artworks

Problems, skills, and techniques exercised by answering the questions: measuring slope, graphing lines, making scale drawings of staircase sections, moving from ramp parameters (rise, run, start, end) to equations to algebraic parameters (slope, intercepts, intersections).

Principal artworks:

staircases

A. Pick a staircase going down and compute its steepness by measuring the rise and run of one step along a wall and computing the slope. Do it over several steps with a partner, using the ring & string to help get a vertical line.
B. Now try the main staircase going to the second floor. The slope will be different at the two side walls and at the center bannister. Why?
C. Several staircases have landings. Does the slope change before and after the landing? Does the direction of the staircase change?

D. The landing will not affect the rise, but if you include it in the run, you get a lower "average" slope for the whole staircase. Is the result a useful measure of the steepness?

Diagonal Freeway

E. How steep is this freeway? Notice there are several lines to consider. They appear to intersect outside the frame, somewhere below and to the left of it. Do they all intersect at the same place? Can you figure out where that would be?

Igbo Door

F. There are numerous lines and line segments in this carving. But there seem to be only three slopes plus vertical. How can you figure out the slopes?

From the Garden of the Chateau

G. There are numerous lines in this painting. They might be wires, they might be rays of light, or they might be elements of the artist's composition meant to suggest something else. If you graph these lines, you may see some that have the same slope but different y-intercepts (and x-intercepts). Or they may have the same y-intercept and different slopes. Can you tell this from equations you construct for them?

Study of Architecture in Florence

H. Parts of the building seem to lie on the same lines. Many of these lines are slanted. Moreover, as in *Diagonal Freeway*, they appear to meet in bunches outside the frame. Sketch a pair that meet, find equations, and see if your estimation of where the meet satisfies both equations. Do this for a different pair that appear to meet in a different place.

Additional artworks:

Feather Tunic The appearance of slanted lines is achieved by staircases, much like on a computer screen. What slopes are attained here?

Terrace overhangs/Mondrianic shadows Seen from the second floor, the huge Terrace overhangs cast shadows on the courtyards below. Some of these shadows make angles that can be traced onto grid paper held up to the window. Compute their slope from the model made on the graph paper.

Shadow Frieze This painting uses short line segments of few slopes to make an apparent covering of the canvas with triangles. But some of the triangles change orientation, so there are quite a few more slopes than in the *Feather tunic*, for example. Give a way to classify the slopes and triangles.

Pl. 10 from the set, Squares with a Different Line Direction in Each Half Square (ImageBase: LeWitt 122 of 464) Only a limited number of slopes are needed to achieve the texture of this piece of art, much like the *Igbo door*. How could you describe the differences?

Lines From Points to Points, plate 4 (ImageBase: LeWitt 458 of 464) Lines abound here, as well as writing about lines. Consider the some of the same questions asked about the works above.

copper skin There are also short line seqments on a slant marked by the features of the skin (holes, bosses). Find as many slopes as you can.

perspective plates In these drawings, you can see lines that meet at a point inside the drawing, but you know they are really parallel in the scene (see more in Story 6), such as lines of trees, edges of roads, and rooflines of buildings. Take two lines that converge and measure their slopes. If they are different, then in the drawing they are not parallel.See if you can find the widest range of slopes of some parallel lines in one of these plates.

Extensions and explorations

A. Support for the perspective studies below (Story 6) can be emphasized here.
B. Staircase studies can lead to practical representations of the Pythagorean Theorem and computing missing parts of right triangles.
C. Staircasing in the *Feather tunic* can lead to a discussion of aliasing and antialiasing in computer graphics.
D. LeWitt's pieces lend themselves to reimplementation and elaboration. Students may find it fun to make this sort of art.
E. In one the *perspective plates*, see if you can find the widest range of slopes of some parallel lines.

Standards and artworks summary

Target standards principally addressed by this story:

17	**AF1.5**	**Represent quant rels graphically; interpret part of graph**
22	**AF3.3***	**Graph linear fcns; note Δy is same for given Δx; rise/run = slope.**
23	**AF3.4***	**Plot quants whose ratios are constant (ft/in); fit line, interpret slope.**
24	**AF4.1***	**Solve 2-step lin eqns & inequalities in 1 var over the rats; interp.**
25	**AF4.2***	**Solve multistep problems involving rdt and dir variation.**

Other target standards addressed by this story:

2	**NS1.2***	**+,*,-,/ rational numbers (ints, fracs, & term dec); (a/b)^m**
15	**AF1.3***	**Simplify num exprs by applying laws of ratl numrs (assoc ++)**
33	MG3.1	Identify, construct c+se geom figs (alts, mp, diag, bisects, circs)
41	**MR1.1**	**Anal. probs from relns, rel/irrel, missing info, patterns**
44	**MR2.3**	**Estimate unknowns graphically; solve by using logic, arith, alg.**
45	**MR2.4**	**Make & test conjectures using both inductive & deductive reasoning.**

Additional standards involved in this story or its extensions:

16	AF1.4	Use algebraic terminology (e.g., va, eqn, term) correctly.
27	MG1.2	Construct and read drawings and models made to scale.
28	MG1.3*	Use measures expressed as rates & prods; use dim anal to check.
35	MG3.3*	Pyth thm and converse; appls; exper verif by measuring.
42	MR1.2	Formulate & justify math conjectures based on a general descrp.
46	MR3.3	Generalize results and strats and apply in novel problem sits.

Main Works:

staircases
Diagonal Freeway
Igbo Door
From the Garden of Chateau
Study of Architecture in Florence

Additional Works:

Feather tunic

Terrace overhangs/Mondrianic shadows
Shadow Frieze
Pl. 10 from the set, Squares with a Different Line Direction in Each Half Square
Lines From Points to Points, plate 4
copper skin
perspective plates

Story 4. Length, surface area, and volume of odd objects

The de Young Museum is filled with oddly shaped objects. Some are architectural, some are craftswork, most are pieces of art. Approaching them mathematically means asking what their underlying geometry is. Sometimes when you understand that, you can figure out their size even though they have a complicated shape. For example, the *Carved mammoth tusk* is hard to measure inside its plastic box, but you can get an estimate for it. With that, you can calculate an estimate of the volume using the cone formula, even though it spirals around. A harder and more interesting problem is to find the smallest size shipping carton; the plastic display box is clearly a loser in that contest. But if you could move the faces of the display cabinet, you could find the smallest box without having to calculate anything.

Math questions related to artworks

Problems, skills, and techniques exercised by answering the questions: application of formulas for length, area, volume to familiar simple shapes and to objects that can be cut into these shapes. Formulas for surface area and volume of spheres, cones, and pyramids are assumed; although they may be beyond the scope of 7th grade standards, they are similar to formulas covered there. Estimation of size for objects that cannot be measured directly.

Principal artworks:
Haida bentwood box

A. The surface area and volume of the box should be easy to calculate after you measure it, using ring&string and a partner to get the decent measurements outside the display case. The lid is also easy. But this ignores the rounded edges; these are the evidence that this amazing box is made from one piece of bent wood.

benches

B. The full bench arrangement uses smaller benches that have the shape of equilateral triangles when seen from above. But they are really pyramids, half sticking up, half sticking down. So the surface area of the top of the bench can be calculated, but not as simply as the floor area it stands above. Give it a try.

Rainy Season in the Tropics

C. The space between the two rainbows is called Alexander's dark band. Figuring the rainbows are half-circles, how would you calculate the area of the dark band?

Raceme

D. A raceme is a certain flower shape, but this shape is clearly something more familiar. Estimate the height and radius of the base, then use the formula to calculate the volume.

3 Machines

E. Use the sphere formula to figure out the volume of a machine's globe and the volume of one gum ball. Say the globe is 9 inches across and a gumball is 1 inch across. You can use this information to figure there are no more than $9^3 = 729$ gumballs; there are fewer because some of the space inside the globe is gum free.

F. Suppose there are 400 gumballs in a machine. How much more surface area do they have than the surface area of the globe (as a difference and a ratio)?

3 Gems

G. The rings of paving stones around the blue-green disk on the floor have different lengths (say the outside circumferences). But the number of stones is always the same. How do the stones have to increase in their length to keep the number uniform as the rings move out from the center disk?

H. The central chamber is roughly a cylinder topped by a hemisphere (better, but harder: a spherical sector). It will be easier to measure the radius of the base than the height of the cylinder, but estimates can then give a good guess on the surface area and volume of the central chamber.

Cocoa pod coffin

I. This object resembles a cylinder with a cone at each end. From that, you can estimate its surface area (how much paint it will need) and its volume. But it is really made from 8 identical pieces that are a little more complicated. How could you measure the surface area of one of these strips?

Storage basket, Cooking basket

J. The first basket is pretty much a cone, so that should be easy. The second basket appears to be two common shapes combined; identifying them is good enough.

Gambling basket

K. What shape is this? It is like slicing a piece off of a sphere. This is called a spherical cap. If you know the height of the basket and the radius of the sphere, you can use a formula to figure out the capacity of the basket.

Carved mammoth tusk

L. You can get a better estimate of the length of the Carved mammoth tusk from the helix-length formula: L = √((3πR/2)2 + H2), where R is the radius of the cylinder the tusk coils around (about 3/4 of a turn) and H is the height of that cylinder (the distance between the ends of the tusk).

Additional artworks:

Hamon Tower There are six floors that twist, bounded by a smoothly curved skin. How can this volume be calculated? It turns out it is the same as an untwisted prism (doesn't work for the surface area). So count the floors at 12 feet each for height and measure the base by measuring the floor of the observation deck and estimating how much farther it is to the copper skin beyond the wood floor. The area of this slice through the tower has the same shape as the wood floor, and that should be easy to see: it is a parallelogram with two angles of 60 degrees.

Ritual oil dish This shape is simple but it escapes simple description. It is *not* a slice off of a cylinder, but try to see why that is a first approximation.

roofs and pentagonal entrance court The entrance court is surrounded by a pentagon of walls above the entry corridor. One angle is observably 60 degrees (as the corner of an equilateral triangle), and the others are obtuse. Given that a pentagon can be built from three triangles, there must be 540 degrees total. What are some good guesses for the two pairs of matching angles?

Rapids Canyon These shapes are not simple prisms, so our guesses of area and volume based on thinking they are will be wrong. But it is a place to start.

Model: Total Reflective Abstraction Some of these shapes are easy to describe as combinations of standard simple shapes. Many are not. But try.

Pierced Monolith Although this has a strange shape, if you know the area of the face of it, you could calculate the volume easily. Why?

Peruvian vessel No calculation, just give a reasonable three or four part combination to describe it.

Geometric Figures within Geometric Figures (ImageBase: LeWitt 25 of 464) These different shapes give practice on computing the area of a standard shape with a simple hole in it.

Extensions and explorations

A. Derive the helix-length formula: $L = \sqrt{(2\pi R)^2 + H^2}$, where R is the radius of the cylinder the helix wraps once around and H is the height of that cylinder (the distance between the ends of the helix. To do this, unwrap a ramp running up the cylinder and notice that Pythagoras' formula gives the length of the ramp once you see the base of the triangle is the circumference of the base circle of the cylinder.
B. Give a better formula for the surface area of the *Cocoa pod coffin*, using parts of cylinders and cones to describe the 8 strips.
C. Tackle the problem of the volumes in *Rapids Canyon.*
D. Tackle the problem of the area of the back face of *Pierced Monolith* (forget the different shape of the blue part on the front).
E. Expand on using triangles to calculate the sum of interior angles of a polygon.
F. Extend the work on *Model: Total Reflective Abstraction.*

Standards and artworks summary

Target standards principally addressed by this story:

29	**MG2.1**	**Use fmls to find perim & area of 2-3D: tri, quadril, circ, prisms, cyls**
30	MG2.2	Estim & compute area of complex 2-3D figs by decomp.
31	MG2.3	Comp perim, sa, vol of 3D objs from rect solids; scaling vol, sa.

Other target standards addressed by this story:

2	**NS1.2***	**+,*,-,/ rational numbers (ints, fracs, & term dec); (a/b)^m**
14	**AF1.2**	**Use order of ops to eval alg exprns such as 3(2x + 5)^2.**
15	**AF1.3***	**Simplify num exprs by applying laws of ratl numrs (assoc ++)**
17	AF1.5	Represent quant rels graphically; interpret part of graph
33	**MG3.1**	**Identify, construct c+se geom figs (alts, mp, diag, bisects, circs)**
34	MG3.2	Coord graphs to plot simple figs; detmn ln, area; trans/refl image
37	MG3.6*	Elems of 3D objs; skew lines; 3-plane intersections
41	**MR1.1**	**Anal. probs from relns, rel/irrel, missing info, patterns**
44	**MR2.3**	**Estimate unknowns graphically; solve by using logic, arith, alg.**
45	**MR2.4**	**Make & test conjectures using both inductive & deductive reasoning.**

Additional standards involved in this story or its extensions:

16	AF1.4	Use algebraic terminology (e.g., va, eqn, term) correctly.
21	AF3.2	Plot 3D vols as fcn of edge length, base edge length.
27	MG1.2	Construct and read drawings and models made to scale.
36	MG3.4*	Conditions for congruence (general); meaning for sides, angles
42	MR1.2	Formulate & justify math conjectures based on a general descrp
43	MR2.1	Use estimation to verify the reasonableness of calculated results.
46	MR3.3	Generalize results and strats and apply in novel problem sits.

Main Works:

- Haida bentwood box
- benches
- Rainy Season in the Tropics
- Raceme
- 3 Machines
- 3 Gems
- Cocoa pod coffin
- Storage basket, Cooking basket
- Gambling basket
- Carved mammoth tusk

Additional Works:

- Hamon Tower
- Ritual oil dish
- roofs and pentagonal court
- Rapids Canyon
- Model: Total Reflective Abstraction
- Pierced Monolith
- Peruvian vessel
- Geometric Figures within Geometric Figures (ImageBase: LeWitt 25 of 464)

Story 5. Lines and planes in space, ruled surfaces, and belts

There are only a few ways two lines can be placed in space: on top of each other, parallel, intersecting, or skew. Two or three planes also have limited ways to run along or into each other. With more lines, everything gets more complicated, but there are some neat curved surfaces that are built from lines, called ruled surfaces. The skin of the Hamon Tower is such a surface. Another kind of ruled surface is a belt, where the lines run across the belt. These too turn up at the de Young.

Math questions related to artworks

Problems, skills, and techniques exercised by answering the questions: how lines and planes interact in 3-dimensional space, recognition and qualitative analysis of ruled surfaces, different ways belts can be connected and run.

Principal artworks:

Untitled (in the Sculpture Garden)

A. This simple sculpture shows some ways lines can lie in space. Can you see all of the ways a pair of lines can be?

copper skin and exterior walls

B. Looking from the Sculpture Garden toward the museum building itself, you can see some surprising intersections of planes, but not all. Which ones are missing? Look elsewhere around the building.

benches

C. Here you can see several different placements of two and three planes. Identify them.

Raceme

D. Cones are ruled surfaces, as are cylinders; planes are obviously ruled, and that means prisms and pyramids are too. Are spheres ruled?

Hamon Tower

E. The vertical struts you can see from the observation floor show the skin is a ruled surface. Can you see them from outside the building?

UW84DC #2

F. Multiple belts form this sculpture. If a belt is given a half-twist before it is joined in a loop, then it is a Moebius surface or band. This is done in industry for even belt wear, because a Moebius band has only one side. Does this sculpture have any one-sided belts in it?

Mill Room

G. Why is the belt twisted in this painting?

Ritual oil dish

H. The surfaces are not only ruled, they are also types of cylinders (the ruling lines are all parallel). Locate the three ruled surfaces.

Additional artworks:

Terrace overhangs/Mondrianic shadows Seen from the second floor, the huge Terrace overhangs cast shadows on the courtyards below. The shadows fall from lines in space; how does that affect the arrangement of lines in the shadows?

Arc A belt sculpture. How many belts? Are the ends free? Are the belts one-sided?

Aurora 2006 This is also a belt sculpture, but now the belt (one? two?) is bent in a way that it is no longer ruled. Find the curves that the lines became.

Hovor II Like *Aurora 2006*, this work is ruled in small areas, but is distorted overall by hanging and by variation in seal sizes. Find some ruled areas and some that are not.

roofs and pentagonal court observe different rulings on these surfaces from the observation deck

Pl. 10 from the set, Squares With A Different Line Direction in Each Half Square This print shows how ruled lines in simple combinations can produce dramatically different textures, a common tool for engravers. Experiment making your own ruled textures.

Belvedere There are many lines in space in this architecture, some parallel, some not. Do they contribute to the impossibility?

Extensions and explorations

A. When *Hovor II* is hung, it can drape like a fabric. So the horizontal and vertical lines become bent into smooth curves (and that happens in fabric, too), one much like its nearest neighbors on either side. This is the key to texture mapping and to realistic cloth in computer graphics.
B. The unusual illusion in *Belvedere* does not seem to have anything to do with lines alone. What makes this building impossible to exist in three-dimensional space?
C. The *Hamon Tower* surface is actually ruled in two directions. From the inside you can see there are linear horizontal supports; these are somewhat visible from outside too. You might think that being ruled in two directions would keep a surface flat. Guess again.
D. A spiral *staircase* (none in the public part of the museum) gives a ramp that is part of a ruled helicoid, a sort of screw. Archimedes' screw is a pump mechanism based on this idea. It can take the form of a double helicoid, and sometimes you see a staircase that is like this too: two people can climb the different parts of it but not meet.

Standards and artworks summary

Target standards principally addressed by this story:

33	**MG3.1**	**Identify, construct c+se geom figs (alts, mp, diag, bisects, circs)**
37	MG3.6*	Elems of 3D objs; skew lines; 3-plane intersections

Other target standards addressed by this story:

41	**MR1.1**	**Anal. probs from relns, rel/irrel, missing info, patterns**
44	**MR2.3**	**Estimate unknowns graphically; solve by using logic, arith, alg.**
45	**MR2.4**	**Make & test conjectures using both inductive & deductive reasoning.**

Additional standards involved in this story or its extensions:

27	MG1.2	Construct and read drawings and models made to scale.
42	MR1.2	Formulate & justify math conjectures based on a general descrp
46	MR3.3	Generalize results and strats and apply in novel problem sits.

Main Works:

Untitled (in the Sculpture Garden)
copper skin and exterior walls
benches
Raceme
Hamon Tower
UW84DC #2
Mill Room
Ritual oil dish

Additional Works:

Terrace overhangs/Mondrianic shadows
Arc
Aurora 2006

Hovor II
roofs and pentagonal court
Pl. 10 from the set, Squares With A Different Line Direction in Each Half Square, Belvedere

Story 6. Parallels, perspective, other projections, and the shining sun

Parallel lines and planes abound in normal buildings. There are plenty in the de Young Museum too, but there are some surprising departures built right into the architecture, for example the Hamon Tower. In making drawings look realistic, parallel lines (like building horizontals and road edges) are drawn to a point, called the vanishing point. This is the heart of creating perspective. Perspective drawing is a type of projection (think of the bulb in a film projector casting rays out in a cone through the film and onto the screen). There is plenty of perspective in paintings at the de Young Museum but there are several other interesting cases of projection too. One is the bright disk cast on the floor in *3 Gems*; it is not an image of the sun, but an image of the oculus, the hole in the ceiling. It is much larger than the blue/green disk on the floor. Is that surprising?

Math questions related to artworks

Problems, skills, and techniques exercised by answering the questions: Geometric and algebraic conditions for parallels; finding vanishing points in perspective drawings; visualizing objects in space; seeing projections and how projections can help us visualize the original objects.

Principal artworks:

staircases

A. For safety, parallel lines and planes with uniform separation govern staircases (they also rule in the sense of Story 5!). Why?

Anti-Mass

B. The charcoal pieces hang on the wires so that the wires stay parallel, even when the charcoal spins in the slight breeze of several breaths. How can this be done?

Diagonal Freeway

C. There is a vanishing point for the freeway. Can you see where it is on the wall? There is also one for the buildings on the steep street in the background; this one is inside the painting, somewhere on the green glass building at the left.

Study of Architecture in Florence

D. There are two obvious vanishing points; one lies outside the painting but on its frame. The other one is on the wall. Find them. Is there a third one? Well, the columns taper toward the top, and their parallel sides do seem to aim together. But if that were so

The Limited

E. Another painting with strong perspective, but a single vanishing point. Where is it? Perspective can explain why the smoke plume looks smaller in the distance. What else can?

From the Garden of Chateau

F. This painting purposely distorts perspective. It harks back to the days just before the Renaissance when artists knew something about parallels appearing to meet in the distance but made it more complicated than it needed to be. See if you can find three violations of perspective here.

Rainy Season in the Tropics

G. Circles can be parallel too. When does that happen?

3 Gems

H. The rings around the central blue/green disk are parallel. Why?
I. At certain times of day, the oculus (the hole in the roof) is projected by the sun on the floor. Because the sun is so far away, its rays are nearly parallel. So the image of the hole on the floor is really the same size as the hole seen along the sunbeams (why? because the oculus is parallel to the floor). What does this say about how large the hole appears when you look up at it?

Asawa wire sculptures

J. These wire sculptures are projected on the walls. When they turn in a slight breeze, the projections can tell more about what the original looks like. How does that happen?

Terrace overhangs/Mondrianic shadows

K. Seen from the second floor, the huge Terrace overhangs cast shadows on the courtyards below. Notice how the projected shadows are differ from the original architectural structures.

Anthropomorphic board

L. The sides of this board are not parallel. But like the staircase, you may be able to find a place to look up at it so that they appear parallel. You are establishing a point of view and a vanishing point that counteract the sides' convergence.

Rapids Canyon

M. Here is an example of geometric forms that you might expect to be prisms with parallel edges and faces but are far from it. Can a different point of view (or camera position) make them look like prisms?

Additional artworks:

Igbo door Here, diamond shapes are stacked in concentric parallel collections, but how do they interact with other parallel collections in the carving?

Girandole mirror This spherical-cap mirror makes interesting distortions. How would you describe them?

benches As noted in Story 5, the wooden benches have many parallel elements. Identify them.

Hamon Tower Although skew lines predominate in the ruled surfaces, each floor is a parallelogram. Can you imagine drawing the tower in perspective?

perspective plates This collection of French perspective drawings shows just about every point of view and vanishing point combination. Describe them.

perspective diagrams The French collection also contains detailed diagrams of how to make perspective drawings and samples of odd objects seen from different points of view. Compare them.

Belvedere The impossibility of the building still permits convincing perspective in the drawing. How does the perspective make the building look even more impossible?

Other World The perspective in this etching drives the confusion by giving a sense of what is up and down. But is there an up and down for the whole picture?

Pl. 10 from the set, Squares With A Different Line Direction in Each Half Square A study in parallel ruling.

Extensions and explorations

A. The main *staircase* to the second floor has a strong antiperspective widening at the top. Can you find a place to stand to make the walls look parallel?

B. The hole in the roof of *3 Gems* is not projected on the floor as a circle (because we are north of the Tropic of Cancer), but as an eccentric ellipse. The minor axis, when it lies on the floor, is the true width of the oculus. Why?

C. In *Study of Architecture in Florence*, is there a third vanishing point? Well, the columns taper toward the top, and their parallel sides do seem to aim together. But if that were so, then columns farther from the middle of the painting would be tilted more, and they are not. The conclusion is that the columns are built tapered and we do not have them represented in perspective. What happens in the real world? How is vertical perspective perceived? Why do painters ignore it?

D. The *girandole mirror* changes angles. Draw a triangle on a piece of paper, then look at it in mirror. If you hold it right, you can get three right angles! Will any triangle work?

E. Among the *Terrace overhangs/Mondrianic shadows* you can see lines that converge to the right in perspective and also converge to the left. Lines that do that have to be curved, don't they? Think about standing in the middle of a pair of railroad tracks; they converge to the right and to the left, but they do not look curved. Explain this.

Standards and artworks summary

Target standards principally addressed by this story:

22	**AF3.3***	**Graph linear fcns; note Δy is same for given Δx; rise/run = slope.**
34	MG3.2	Coord graphs to plot simple figs; detmn ln, area; trans/refl image

Other target standards addressed by this story:

33	**MG3.1**	**Identify, construct c+se geom figs (alts, mp, diag, bisects, circs)**
37	MG3.6*	Elems of 3D objs; skew lines; 3-plane intersections
41	**MR1.1**	**Anal. probs from relns, rel/irrel, missing info, patterns**
44	**MR2.3**	**Estimate unknowns graphically; solve by using logic, arith, alg**
45	**MR2.4**	**Make & test conjectures using both inductive & deductive reasoning.**

Additional standards involved in this story or its extensions:

16	AF1.4	Use algebraic terminology (e.g., va, eqn, term) correctly.
27	MG1.2	Construct and read drawings and models made to scale.
36	MG3.4*	Conditions for congruence (general); meaning for sides, angles
42	MR1.2	Formulate & justify math conjectures based on a general descrp
46	MR3.3	Generalize results and strats and apply in novel problem sits.

Main Works:

- staircases
- Anti-Mass
- Diagonal Freeway
- Study of Architecture in Florence
- The Limited
- From the Garden of Chateau
- Rainy Season in the Tropics
- 3 Gems
- Asawa wire sculptures
- Terrace overhangs/Mondrianic shadows
- Anthropomorphic board
- Rapids Canyon

Additional Works:

- Igbo door
- Girandole mirror
- benches
- Hamon Tower
- perspective plates
- perspective diagrams
- Belvedere
- Other Worlds
- Pl. 10 from the set, Squares With A Different Line Direction in Each Half Square

Story 7. Uniformity, symmetry, chaos, confusion, and the feeling of being lost

Sometimes art is all about uniformity, but usually it works on breaking up the order and introducing some element of diversity, even chaos, sometimes confusion. The de Young Museum features art lying across this entire range. But now we focus on the extremes: great sameness and great difference. The mathematics of sameness is called symmetry, the geometry of chaos is called fractal geometry, and the logic of confusion is called paradox. Any of these extremes can give the sense of being lost. If everything is the same (say, you are in the middle of *Strontium*), every place looks alike (but not every direction). If you are on the crack in *Drawn Stone* or on a mountain in *Prometheus Bound*, it looks rough, but pretty much like any other place on the crack or peak. If you are looking at *Other World*, you might be looking up, or down, or sideways. People talk about losing themselves in art; in these cases, the loss is real.

Math questions related to artworks

Problems, skills, and techniques exercised by answering the questions: finding symmetry and uniformity; finding variety and scaling roughness; finding impossibility and perplexity.

Principal artworks:
Strontium

A. This work has symmetry: one place is much the same as many nearby places. In particular, there is no difference between one ball and its neighbors of the same size. The picture can shift left or right a column, or up or down a row, but it "resists" being turned sideways because the rows of large balls are different than the columns of small large (same idea for the small balls, too). This type of symmetry is called *translation symmetry*. Can you see some other translation symmetries in the art?

benches

B. The benches have rotation symmetry. How much do you have to turn a set of 6 benches till it looks like the original (ignore the woodgrain)? How about in the other direction?

3 Gems

C. There is rotation symmetry on the floor, and chaos in the mixture of colors in the central disk. If the benches continued all the way around, and the archways were closed, can you find other rotation symmetries?

pavers in courtyards

D. Many of the tile floors here have strong symmetry, but some do not. Even when there are many parallel rows, the tiles in a row may have different lengths. In the front courtyard the symmetry is broken by occasional double-wide tiles. Are there any edge lines that go wall to wall?

Shadow Frieze

E. There are symmetries that slide the shapes (ignoring the colors) up to the right and up to the left, plus others we have already seen. Up close, there is a lot of confused color. At a distance, both this symmetry and this chaos disappear into a coherent picture of draped cloth, which is paradoxical. Can you find a place to stand where all these extremes are in balance?

Vessel

F. The twisty curves decorating this jar are regular but very complicated. Can you draw what it would look like if one of these curves made even more (and smaller) twists?

Drawn Stone

G. The crack is a scaling fractal, a kind of roughness that is has the same wiggliness from the second floor window, standing above it, or getting your face close to it. But it is not completely random, for it runs fairly straight across the tiles then turns in order to pass through the benches. Is it a loop? Or can you find the ends? What percent of the tiles does it break? What percent of the benches? How in the world could the artist crack stone in such a controlled way?

Prometheus Bound

H. Mountains are fractals: a small part of mountain looks like a miniature mountain. In this painting, if you could not tell from the sky and the color, could you tell much difference between the rock Prometheus is on and the peak in the background?

Asawa wire sculptures

I. Some of these wire sculptures are fractals, especially when they are branching; a small branch looks like a large branch. See if you can find other places where a small part looks like the whole thing.

copper skin

J. Many sheets of the dimpled and pierced copper sheeting on the building are chaotic in their choice of size and shape of the bosses and holes. People say they were designed on a computer, but that just raises the question of what the computer program did to choose these varied designs. See how many varieties of these sheets you can find.

Button blanket

K. Is the frog in the middle from a top view, or are there two frogs on two sides in profile? Are they all frogs? What does it mean to be shown both the inside and outside of something at the same time?

Additional artworks:

The Limited The smoke puffs, like many clouds, form another sort of fractal: a puff can look like a small version of the whole plume. See if you can find a large part and a small part that look most alike.

Belvedere The paradox is that one can draw this convincingly on two-dimensional paper, but you can see that it cannot be built realistically in three dimensions. Why?

Conservation Chair This resembles impossible waterfalls by Escher, but here the paradox is not the impossibility of the Belvedere, but the suggestion that the water moves up the pipe on its own accord. Why?

Print Gallery Inside becomes outside in this paradoxical world. What should be at the blank place in the center? A famous mathematician has filled it in (several different ways!).

Other World The paradox depends on a shift in gravity. Where and how?

Whirlpools Is this symmetric or chaotic? Much more the former. Ignoring the color, there is a big rotation symmetry. It is true that a part of the spiral looks like a miniature of the whole spiral, but that only happens in one place, so it is not rough enough to be a fractal.

Geometric Figures within Geometric Figures (ImageBase: LeWitt 25 of 464) This is a new kind of uniformity, without the symmetry seen elsewhere. How would you describe the sameness of this drawing?

Lines from Point to Point, pl. 4 (ImageBase: LeWitt 458 of 464) This is an example of identical elements (suggesting symmetry) being assembled in a chaotic arrangement (suggesting fractality).

Extensions and explorations

A. There are symmetries besides translation and rotation. A butterfly has reflection symmetry. Can you see reflection symmetries in your face? in the benches (not so easy, but it is there, three times!)? elsewhere at the de Young Museum?

B. The twisty curves decorating the *Peruvian vessel* can be iterated by replacing the longer line segments with reduced versions of the "bay." Sometimes this can produce a space-filling curve. Try drawing the next iteration.

C. *Drawn Stone* is like a coastline. If you try to measure it with smaller and smaller units, it takes more and more of them, as you would expect. But the length gets longer and longer. Read up on this scaling property of fractals, and see if you can measure the fractal dimension of the crack experimentally.

D. Make a fractal collage similar to *Lines from Point to Point*, but go further by using a copy machine to make the text different sizes.

E. Build a simplified model of the *Belvedere* (without all the domes and decoration) that looks convincing from one angle.
F. Investigate Lenstra's exploration of the *Print Gallery* and how it is like a box of Droste cocoa.

Standards and artworks summary

Target standards principally addressed by this story:

31	MG2.3	Comp perim, sa, vol of 3D objs from rect solids; scaling vol, sa.
33	**MG3.1**	**Identify, construct c+se geom figs (alts, mp, diag, bisects, circs)**

Other target standards addressed by this story:

34	MG3.2	Coord graphs to plot simple figs; detmn ln, area; trans/refl image
41	**MR1.1**	**Anal. probs from relns, rel/irrel, missing info, patterns**
44	**MR2.3**	**Estimate unknowns graphically; solve by using logic, arith, alg.**
45	**MR2.4**	**Make & test conjectures using both inductive & deductive reasoning.**

Additional standards involved in this story or its extensions:

21	AF3.2	Plot 3D vols as fcn of edge length, base edge length.
27	MG1.2	Construct and read drawings and models made to scale.
42	MR1.2	Formulate & justify math conjectures based on a general descrp
46	MR3.3	Generalize results and strats and apply in novel problem sits.

Main Works:

Strontium
benches
3 Gems
pavers in courtyards
Shadow Frieze
Vessel
Drawn Stone
Prometheus Bound
Asawa wire sculptures
copper skin
Button blanket

Additional Works:

The Limited
Belvedere
Conservation Chair
Print Gallery
Other World
Whirlpools
Geometric Figures within Geometric Figures (ImageBase: LeWitt 25 of 464)
Lines from Point to Point, pl. 4